Model-based Dynamic Fractional-order Sliding Mode Controller Design for Performance Analysis and Control of a Coupled Tank Liquid-level System

Hayriye Tugba SEKBAN[1], Kaan CAN[2], Abdullah BASCI[2]
[1]*Department of Electronics and Automation, Bayburt University, Bayburt, Turkey*
[2]*Department of Electrical and Electronics Engineering, Ataturk University, Erzurum, 25240, Turkey*
abasci@atauni.edu.tr

***Abstract*—In this paper, a model-based dynamic fractional-order sliding mode controller (FOSMC) is designed and implemented to a coupled tank experimental setup for controlling the liquid level. First, a model-based dynamic sliding-mode controller is designed by using the dynamic equations of a vertically positioned coupled tank system. Then, the sliding surface of the sliding-mode controller is defined in fractional order so that the designed controller can make better water level tracking. The liquid level control of the system is realized in two different steps. In the first step, the water level of the upper tank is controlled by a pump and in this application the bottom tank is not considered. In the second step, the water level of the bottom tank is controlled with upper tank's output water. In addition, a model-based dynamic sliding mode controller (SMC) is also applied to the system to show the performance of the proposed controller in terms of robustness to disturbances, reference tracking and error elimination capability. Experimental results show that the proposed controller reduces the reference tracking error by 3.68% and 10.17% for the upper tank and 17.07% for the bottom tank when compared to the SMC, and the control signal contains more chattering than the SMC.**

***Index Terms*—fractional calculus, level control, nonlinear control systems, process control, sliding mode control.**

I. Introduction

Liquid level control is one of the most important industrial process steps. In the last decade, the liquid level control systems have been often used in different areas such as petro chemistry and medicine industries, water treatment and energy plants etc., where the controller must be very sensitive to parameter changes and need to be robust to disturbances. Moreover, liquid level controller carries out some complex missions such as pumping liquid to other tanks, adjusting required liquid level and storage of the desired level of the liquid in tanks. In this process, it is hard to provide exact liquid quantity and regulate the flow rate between tanks. In addition, an important problem for this kind of systems is that they have a nonlinear behaviour due to irregular flow rate that changes with respect to time. To overcome these difficulties and control liquid level system in a stable way, many control strategies have been proposed in control engineering and related research areas. First, the conventional PI, PD and PID controllers are used to control tank systems [1-2]. Sekban et al. [3], have designed a fractional-order PI controller to enhance the performance of the integer-order PI controller performance. The experimental results show that the proposed FOPI controller has shown better reference tracking performance than PI controller. However, since nonlinear system dynamics and parameter adjustment are required, the liquid level control system should be controlled with nonlinear controllers. Therefore, some studies are presented with solutions that overcome the problems concerned on liquid level control systems.

Sekban et al. [4], have proposed a model-based dynamic SMC to control water level of a coupled tank system. Also, they have compared SMC with PI controller to show the superiority of SMC. Experimental results have indicated that the designed control algorithm has had satisfying control performance. Can et al. [5], have designed a backstepping controller as a nonlinear controller based on the system dynamics and also, the proposed control algorithm has compared PI controller under real-time experiment. The experimental results have shown the superiority of the backstepping controller. Boonsrimuang et al. [6], have applied PI – Model Reference Adaptive Controller (MRAC) to coupled tank liquid level system and the results indicate that the designed model based controller is good at eliminating steady state error and dealing with parameter uncertainties occurred in the system dynamics. The authors of the study [7], have used fuzzy and conventional PID controllers to realize level control of coupled tank system. The simulation results show that fuzzy controller is better than PID controller in terms of overcoming parameter uncertainties and eliminating steady state error occurring in the system. Başçi and Derdiyok [8], have presented adaptive fuzzy controller to control coupled tank liquid level system that is compared with conventional PI controller in terms of reference tracking performance, rising-settling time and error elimination success. The experimental results indicate that adaptive fuzzy controller showed better performance with respect to parameter changes, has lower settling time and good at tracking reference input when it is compared with PI controller.

As another nonlinear controller, sliding mode controller (SMC), is proposed to control coupled tank liquid level system [9-15]. Abbas et al. [16], have designed SMC to apply coupled tank liquid level system and analysed the controller mathematically to compare with PID controller.

Digital Object Identifier 10.4316/AECE.2020.03011

The simulation results show that SMC has better trajectory tracking performance for different reference inputs. Efe and Kasnakoğlu [17], have proposed fractional order differentiation and integration to increase system performance where system dynamics needs to be flexible to changes. Also, they have used fractional calculus with adaptive sliding mode controller to achieve and design better performing controller as well.

In this paper, a model-based dynamic FOSMC is compared with SMC in real-time application on a coupled tank liquid level system to show its reducing effect on chattering phenomena that causes bad effect on the system dynamics and robustness against to parameter uncertainties and disturbances. The experimental results show that FOSMC is good at coping with undesired system behaviours, providing enough flexibility, is robust to parameter variations as well as to external disturbances. Also, the FOSMC has better time-varying reference inputs tracking performance when it is compared with SMC.

II. The Coupled Tank System

A. *Modelling of the coupled tank liquid level system*

The coupled tank system consists of a pump with a water basin and two tanks. Tanks are located vertically on a platform. The pump feeds the tank 1 and the bottom tank, which is called as tank 2, is fed by the output of the upper tank. The coupled tank system is used in two different configurations: configuration #1 and configuration #2 respectively. Our study focused on the level control of the top tank in configuration #1 and level control of bottom tank in configuration #2.

B. *Single tank model (configuration* #1)

Single tank system consisting of the top tank is shown in Fig. 1. It is reminded that in configuration #1, the pump feeds into tank 1 and that tank 2 is not considered at all. Therefore, the input to the process is the voltage to the pump and its output is the water level in tank 1.

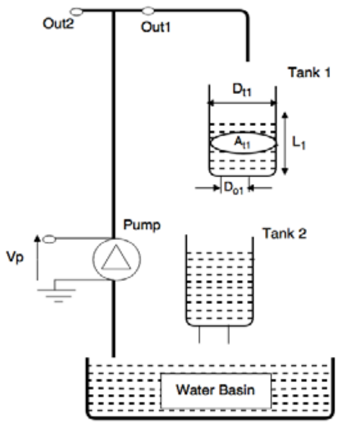


Figure 1. Single tank model

The mathematical model of the single tank system determined by relating the volumetric inflow rate f_{i1} into tank 1 and the outflow rate f_{o1} leaving through the hole at the tank 1 bottom. The volumetric inflow rate and the outflow rate to tank 1 can be expressed as [4,18].

$$f_{i1} = \eta u(t) \tag{1}$$

$$f_{o1} = A_{o1} V_{o1} \tag{2}$$

where A_{o1} is the outlet cross sectional area (cm^2), V_{o1} is the tank 1 outflow velocity (cm/s), $\eta = K_p / A_{t1}$ is constant, K_p is the pump volumetric flow constant $(\frac{cm^3}{s}/V)$ and $u(t) = V_p$ is the actual pump input voltage $(volt)$. The outflow velocity by using Bernoulli's equation

$$V_{o1} = \sqrt{2}\sqrt{gL_1} \tag{3}$$

where g is the gravitational constant on earth. As a remark, the cross-section area of tank 1 outlet hole can be calculated by,

$$A_{o1} = \frac{1}{4}\pi D_{o1}^{\,2} \tag{4}$$

In the Eq. (4) D_{o1} is the tank 1 outlet diameter. Using Eq. (3) and (4) the outflow rate from tank 1 given in Eq. (2) becomes,

$$f_{o1} = A_{o1}\sqrt{2}\sqrt{gL_1} \tag{5}$$

Moreover using the mass balance principle for tank 1, we obtain the following first-order differential equation in L_1,

$$A_{t1}\left(\frac{dL_1}{dt}\right) = f_{i1} - f_{o1} \tag{6}$$

where A_{t1} is the cross-section area of tank 1. Substituting Eq. (1) and (5) into Eq. (6) and rearrange the equation the following form for the tank 1 system can be obtained.

$$\frac{dL_1}{dt} = \frac{KpVp - A_{o1}\sqrt{2}\sqrt{gL_1}}{A_{t1}} \tag{7}$$

C. *Coupled tank model (configuration* #2)

A schematic of the coupled tank plant is depicted in Fig. 2. In configuration #2 the pump feeds into tank 1, which in turn feeds into tank 2.

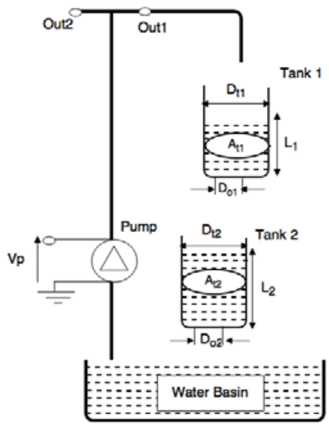


Figure 2. Coupled tank model

As far as tank 1 is concerned, the same equation as the ones previously developed in section (B) is applied. However, the water level equation of motion in tank 2 still needs to be derived. In the coupled tank, the system states are the level L_1 in tank 1 and the level L_2 in tank 2. The outflow rate from tank 2 can be expressed as [4,18];

$$f_{o2} = A_{o2} V_{o2} \tag{8}$$

Tank 2 outflow velocity by using Bernoulli's equation,

$$V_{o2} = \sqrt{2}\sqrt{gL_2} \tag{9}$$

As a remark, the cross-section area of tank 2 outlet hole can be calculated by,

$$A_{o2} = \frac{1}{4}\pi D_{o2}{}^{2} \tag{10}$$

Using Eqs. (9) and (10) the outflow rate from tank 2 given in Eq. (8) becomes

$$f_{o2} = A_{o2}\sqrt{2}\sqrt{gL_2} \tag{11}$$

Using Eq. (5) as inflow rate or tank 2,

$$f_{i2} = A_{o1}\sqrt{2}\sqrt{gL_1} \tag{12}$$

Moreover using the mass balance principle for tank 2, we obtain the following first-order differential equation in L_2,

$$A_{t2}\left(\frac{dL_2}{dt}\right) = f_{i2} - f_{o2} \tag{13}$$

Substituting Eq. (12) and (11) into Eq. (13) and rearrange the equation the following form for the tank 2 system can be obtained.

$$\frac{dL_2}{dt} = \frac{-A_{o2}\sqrt{2}\sqrt{gL_2} + A_{o1}\sqrt{2}\sqrt{gL_1}}{A_{t2}} \tag{14}$$

III. Controller Design

A. Fractional order calculation

The fractional calculus has been used for a long time. Although the fractional calculus has a long past, it has been used in control engineering or related research areas in recent history [19]. Today, with the improvement of technology, computers are very fast and capable of calculating very complex equations that have high order terms with integer or fractional sensitively.

For fractional order terms of differentiation or integration, they can be represented as ${}_aD_t^r$ clearly while doing fractional order calculation, and also a and t are the lower and upper limits of calculation where $r \in R$. For the described form of continuous integro-differential representation can be written as below [20,21].

$${}_aD_t^r = \begin{cases} \dfrac{d^r}{dt^r} & r \rangle\, 0 \\ 1 & r = 0 \\ \int_a^t (dt)^{-r} & r \langle\, 0 \end{cases} \tag{15}$$

The Grünwald-Letnikov (GL), Riemann-Louville (RL) and Caputo methods are well-known methods to use in fractional order calculations [22]. The methods mentioned are shown below respectively [22]. The definition of the GL method can be expressed [22];

$${}_aD_t^r f(t) = \lim_{h\to 0} h^{-r} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{r}{j} f(t-jh) \tag{16}$$

and here $\left[\frac{t-a}{h}\right]$ term is an integer term. The RL method can be described for $n-1 \langle r \langle n$;

$${}_aD_t^r f(t) = \frac{1}{\Gamma(n-r)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{r-n+1}} d\tau \tag{17}$$

where $\Gamma(.)$ is the function of Gamma. For $n-1 \langle r \langle n$, the Caputo method can be written given in Eq. (18) [23,24].

$${}_aD_t^r f(t) = \frac{1}{\Gamma(n-r)} \int_a^t \frac{f^n(\tau)}{(t-\tau)^{r-n+1}} d\tau \tag{18}$$

The initial conditions for fractional order derivative equations are the same structure with integer order derivative equations' in Caputo's method [20]. Although the fractional order calculation cannot be computed exactly, some approaches have been derived. The main idea of the proposed approaches on fractional order calculation is to achieve its integer order approximation to realize as a computing method physically [20]. The systems that are represented in Laplace transformation with fractional order, can be computed by using some methods such as Carlson, Matsuda, Tustin, Simpson, Crone [20-25]. These approximation methods are depending on series expansion of the Laplace operator.

B. Sliding mode controller design

B.1 SMC design for configuration #1

To design SMC, some assumptions must be determined [26]. First, the flow rates cannot be negative $f \geq 0$ and with some assumptions the following equations can be written.

$$\frac{A_{o1}\sqrt{2g}}{A_{t1}} = k_1, \frac{A_{o2}\sqrt{2g}}{A_{t2}} = k_2, \quad k_1 = k_2 = k \tag{19}$$

If writing these assumptions in place of the coupled tank system, the dynamic model equations can be obtained from the following equations [26].

$$\dot{L}_1 = -k\sqrt{L_1} + \eta u \, , \dot{L}_2 = k\sqrt{L_1} - k\sqrt{L_2} \tag{20}$$

Defining a sliding surface $s(t)$ as [9,27,32],

$$s = \lambda e_1 + \dot{e}_1 \tag{21}$$

where λ is a positive constant, $e_1 = L_{1r} - L_1$ is the error that is expressed as the difference between reference and measured value of water level. Taking time derivative of the s function, the following equation can be obtained.

$$\dot{s} = \lambda\dot{e}_1 + \ddot{e}_1 \tag{22}$$

After taking the second order derivative of the e_1 and replacing it in Eq. (22), the following equation can be obtained.

$$\dot{s} = \lambda\dot{e}_1 + (\ddot{L}_{1r} - \ddot{L}_1) \tag{23}$$

where,

$$\ddot{L}_1 = \frac{-k}{2\sqrt{L_1}}\left(-k\sqrt{L_1} + \eta u\right) \tag{24}$$

To design a control strategy based on the system dynamics, a well-defined positive-definite Lyapunov candidate function can be selected as given below [28].

$$V = \frac{1}{2} s P s^T > 0 \tag{25}$$

where $P = P^T > 0$. Taking time derivative of Eq. (25), Eq. (26) can be obtained.

$$\dot{V} = s^T P \dot{s} \tag{26}$$

Here, to guarantee the stability of the system, the time derivative of the Lyapunov candidate function must be negative definite. To provide this, Eq. (26) can be equal to Eq. (27) with the assumption given below.

$$\dot{V} = -s^T G\, sign(s) = s^T P \dot{s} < 0 \tag{27}$$

where G is a design parameter and can be selected properly. If Eq. (23) is rewritten in Eq. (27), Eq. (28) is obtained.

$$-Gsign(s) = P((\ddot{L}_{1r} - \ddot{L}_1) + \lambda(\dot{L}_{1r} - \dot{L}_1)) \quad (28)$$

If Eq. (28) is rearranged, then Eq. (29) can be achieved.

$$-Gsign(s) = P(\ddot{L}_{1r} - \frac{k^2}{2} + \frac{k\eta}{2\sqrt{L_1}}u) + P\lambda(\dot{L}_{1r} + k\sqrt{L_1} - \eta u) \quad (29)$$

Assuming that P is an identity matrix and then, the control signal u is left alone, the control signal for configuration #1 can be achieved as given in Eq. (30).

$$u = \frac{2\sqrt{L_1}}{P\eta(2\lambda\sqrt{L_1} - k)}\left(P(\ddot{L}_{1r} + \lambda\dot{L}_{1r} + \lambda k\sqrt{L_1} - \frac{k^2}{2}) + Gsign(s)\right) \quad (30)$$

B.2 SMC design for configuration #2

In the same manner, the control law has been obtained for the level control of tank 2 considering the dynamics of the coupled tank system given in Eqs. (8-14) and Eq. (22) as well. Considering the previous configuration, we can define the system model for configuration #2 as given below in which the output is defined as L_2 [9,26].

$$\dot{L}_1 = -k\sqrt{L_1} + \eta u \,, \dot{L}_2 = k\sqrt{L_1} - k\sqrt{L_2} \quad (31)$$

Then, to design SMC for configuration #2, a sliding surface can be defined as in Eq. (21) for $e_2 = L_{2r} - L_2$ as given below [9,27,32].

$$s = \lambda e_2 + \dot{e}_2 \quad (32)$$

Taking time derivative of Eq. (32), the following equation can be obtained.

$$\dot{s} = \lambda\dot{e}_2 + \ddot{e}_2 \quad (33)$$

When the above definition of e_2 is used in Eq. (33), the following equation is obtained.

$$\dot{s} = \lambda\dot{e}_2 + (\ddot{L}_{2r} - \ddot{L}_2) \quad (34)$$

where,

$$\ddot{L}_2 = \frac{-k}{2\sqrt{L_2}}\left(k\sqrt{L_1} - k\sqrt{L_2}\right) \quad (35)$$

Rewriting Eqs. (31) for $\dot{L}_2$ and (35) into Eq. (34), and defining a positive-definite Lyapunov candidate function as realized in Eq. (25) and using its negative-definite differentiation as well, the desired control signal for configuration #2 can be derived as given below.

$$u = \frac{1}{P\lambda\eta}\begin{pmatrix} P\lambda(\dot{L}_{2r} + \dot{L}_1 + k\sqrt{L_2}) + P(\ddot{L}_{2r} + \frac{k^2}{2}\sqrt{\frac{L_1}{L_2}} - \frac{k^2}{2}) \\ + Gsign(s) \end{pmatrix} \quad (36)$$

C. Fractional order sliding mode controller design

C.1 FOSMC design for configuration #1

In this section, FOSMC is designed depending on the Eq. (21) that was used to design SMC. First of all, a sliding surface with fractional order derivative can be defined as [22],

$$s = \lambda e_1 + {}_aD_t^r e_1 \,,\ \lambda \in R^+, r \in (0,1) \quad (37)$$

where λ is the gain of sliding surface, and ${}_aD_t^r(\cdot)$ is the fractional calculus operator. Taking derivative of Eq. (37) with respect to time, the following equation is obtained.

$$\dot{s} = \lambda\dot{e}_1 + {}_aD_t^{r+1} e_1 \quad (38)$$

Defining a positive-definite Lyapunov candidate function [27],

$$V = \frac{1}{2}sPs^T > 0 \quad (39)$$

where $P = P^T > 0$ and then, taking time derivative of the Eq. (39), Eq. (40) can be obtained.

$$\dot{V} = s^T P\dot{s} \quad (40)$$

To proof the stability of the control algorithm, the time derivative of the Lyapunov candidate function must be negative-definite that is expressed as given below.

$$\dot{V} = -s^T Gsign(s) = s^T P\dot{s} < 0 \quad (41)$$

where G is a design parameter and can be selected properly. From Eq. (38), the fractional-order sliding surface can be written into Eq. (41) as given below.

$$-Gsign(s) = P({}_aD_t^{r+1}(L_{1r} - L_1) + \lambda(\dot{L}_{1r} - \dot{L}_1)) \quad (42)$$

To achieve the needed control signal equation, the states of the system are rewritten into Eq. (42) and then, Eq. (43) can be obtained.

$$-Gsign(s) = P\,{}_aD_t^{r+1}e_1 + P\lambda(\dot{L}_{1r} + k\sqrt{L_1} - \eta u) \quad (43)$$

Assuming that P is an identity matrix and then, the control signal u is left alone, the fractional-order control signal for configuration #1 can be achieved as given in Eq. (44).

$$u = \frac{1}{P\lambda\eta}\left(P\,{}_aD_t^{r+1}e_1 + Gsign(s) + P\lambda(\dot{L}_{1r} + k\sqrt{L_1})\right) \quad (44)$$

C.2 FOSMC design for configuration #2

To design the FOSMC for configuration #2, the Eq. (33) is taken into account. Then, a fractional order sliding surface can be defined as follow [22].

$$s = \lambda e_2 + {}_aD_t^r e_2 \,,\ \ \lambda \in R^+, r \in (0,1) \quad (45)$$

Taking time derivative of Eq. (45), Eq. (46) is achieved.

$$\dot{s} = \lambda\dot{e}_2 + {}_aD_t^{r+1} e_2 \quad (46)$$

To obtain a fractional-order control signal for configuration #2, the same Lyapunov candidate function is taken into account as in Eqs.(25) -(39),

$$V = \frac{1}{2}sPs^T > 0 \quad (47)$$

and its differentiation of with respect to time,

$$\dot{V} = s^T P\dot{s} \quad (48)$$

can be obtained. From Eqs. (31) and (41), the following expression can be obtained.

$$-Gsign(s) = (P\,{}_aD_t^{r+1}e_2 + P\lambda(\dot{L}_{2r} - \dot{L}_2)) \quad (49)$$

For more clear expression of Eq. (49), it can be expressed as given below.

$$-Gsign(s) = (P\,{}_aD_t^{r+1}e_2 + P\lambda\dot{L}_{2r} + P\lambda\dot{L}_1 + P\lambda k\sqrt{L_2} - P\lambda\eta u) \quad (50)$$

Finally, the control signal can be obtained for configuration #2 as given below:

$$u = \frac{1}{P\lambda\eta}\begin{pmatrix} PD^{r+1}e_2 + Gsign(s) + P\lambda\dot{L}_{2r} + \\ P\lambda k\sqrt{L_2} + P\lambda\dot{L}_1 \end{pmatrix} \quad (51)$$

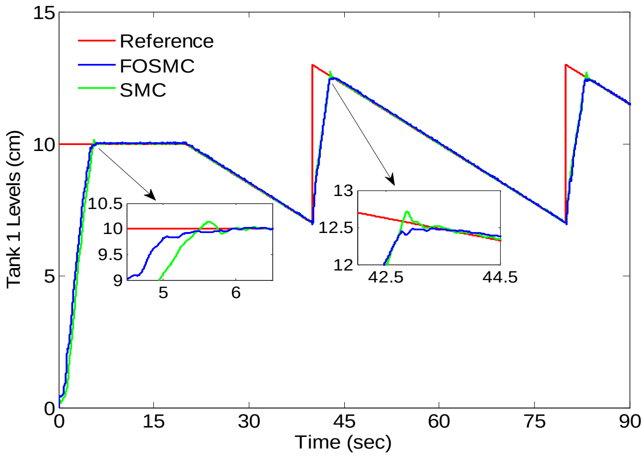
which is fractional order sliding mode control signal of the tank 2, where *Gsign(s)* is switching control function, *G* is the switching gain and *sign(s)* can be expressed as given below [28-31].

$$sign(s) = \begin{cases} 1 & s \rangle\ 0 \\ 0 & s = 0 \\ -1 & s \langle\ 0 \end{cases} \tag{52}$$

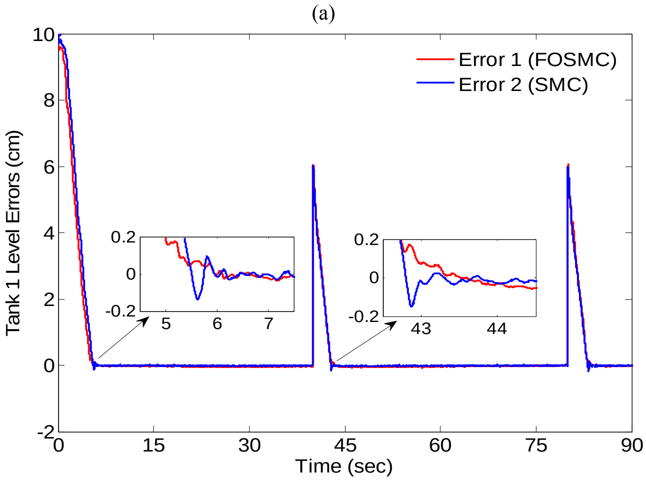
IV. Experimental Results

In this section, the experimental results are presented for both controllers. During the experiment for configuration #1, the G value is determined as 40 and λ is determined as 20 for both controllers. Also, the fractional operator r is taken as 0,6.

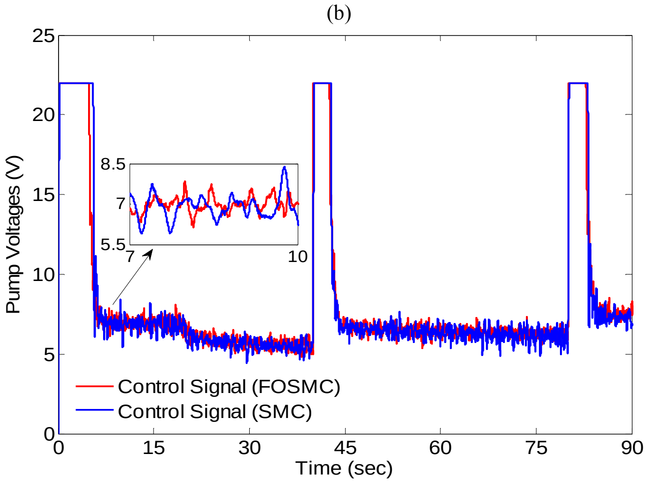
After determining the optimal parameters, the obtained experimental results are given in Fig. 3 (a)-(c) and Fig. 4 (a)-(c) for step + sawtooth and step + square reference signals, respectively. In Fig. 3 (a), the step + sawtooth reference tracking results are given for both controllers. From the obtained results, FOSMC has small rise time as well as less overshoot/undershot level for the step part of the reference signal. When the sawtooth reference signal is applied, although both controllers have nearly the same rise time, SMC has bigger overshot when it compared with the FOSMC. In addition, the obtained error levels presented in Table 1 shows that FOSMC has shown 3.68% better trajectory tracking performance than SMC throughout the step and time-varying part of the reference signal.



(a)

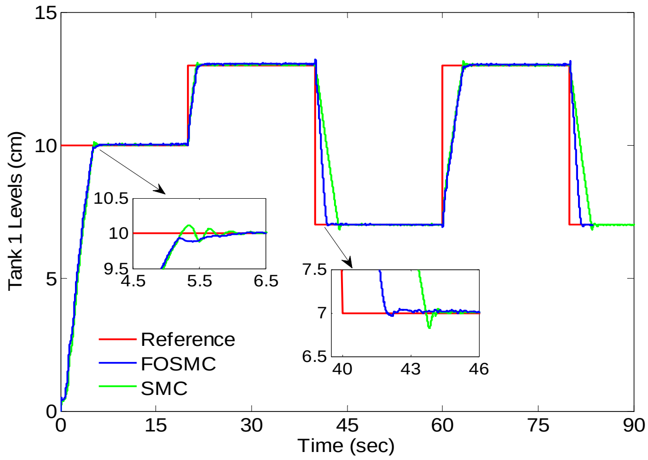


(b)

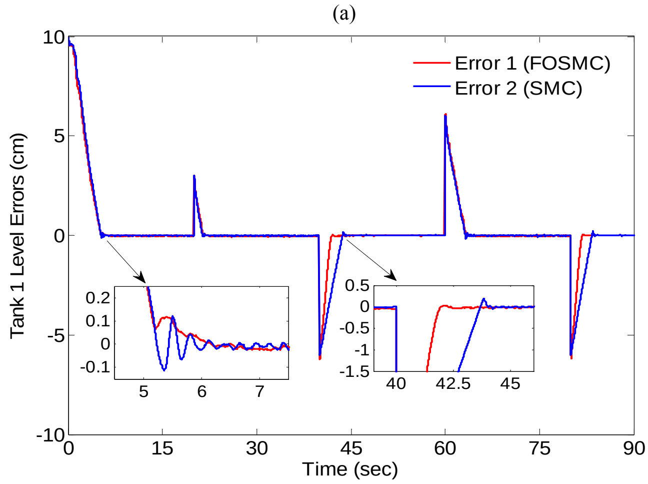


(c)

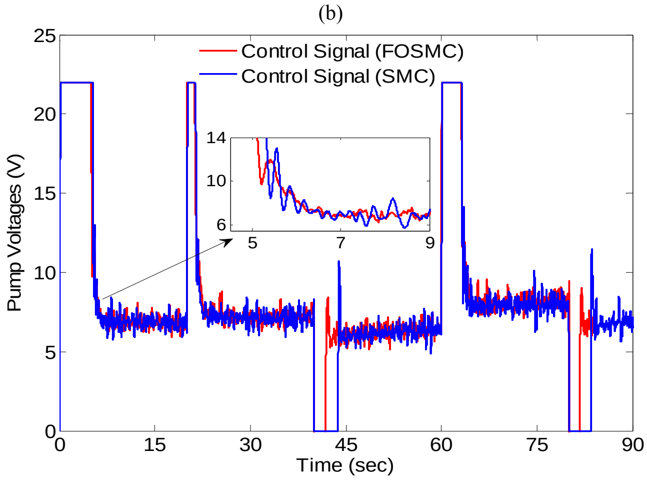
Figure 3. The FOSMC and SMC experimental results for configuration #1 under step + sawtooth reference signal



(a)



(b)



(c)

Figure 4. The FOSMC and SMC experimental results for configuration #1 under step + square reference signal

When the Fig. 3 (c) is analysed, the proposed controller and SMC have produced similar control signal forms that causes similar control performance for configuration #1 under step + sawtooth reference signal.

In the second experiment seen in Fig. 4 (a)-(c), the step + square reference signal is used to show the responses of both controllers according to sudden changes in a period. From the Fig. 4 (a), both controllers have almost the same rise time, whereas SMC has same oscillations while tracking the reference signal. Also, FOSMC has better rise time and settling time between 40 and 46 seconds when the sudden change is applied. In addition, in Fig. 4 (b), FOSMC has better error elimination capability and has given fast response to the sudden changes than SMC as seen in the Table 1. Also, in Fig. 4 (c), both controllers have similar control signal forms, whereas the proposed controller has shown better trajectory performance than the SMC result.

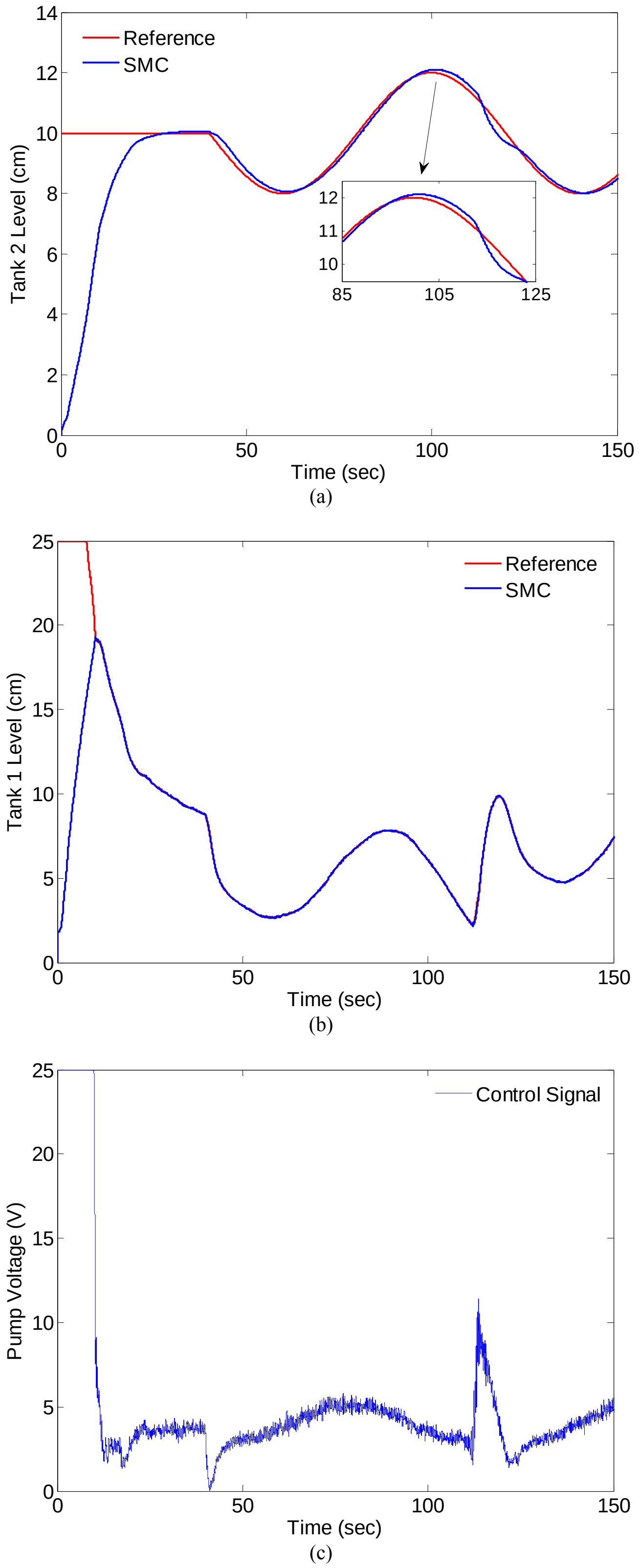


Figure 5. The SMC experimental results for configuration #2 under step + sinusoidal reference signal

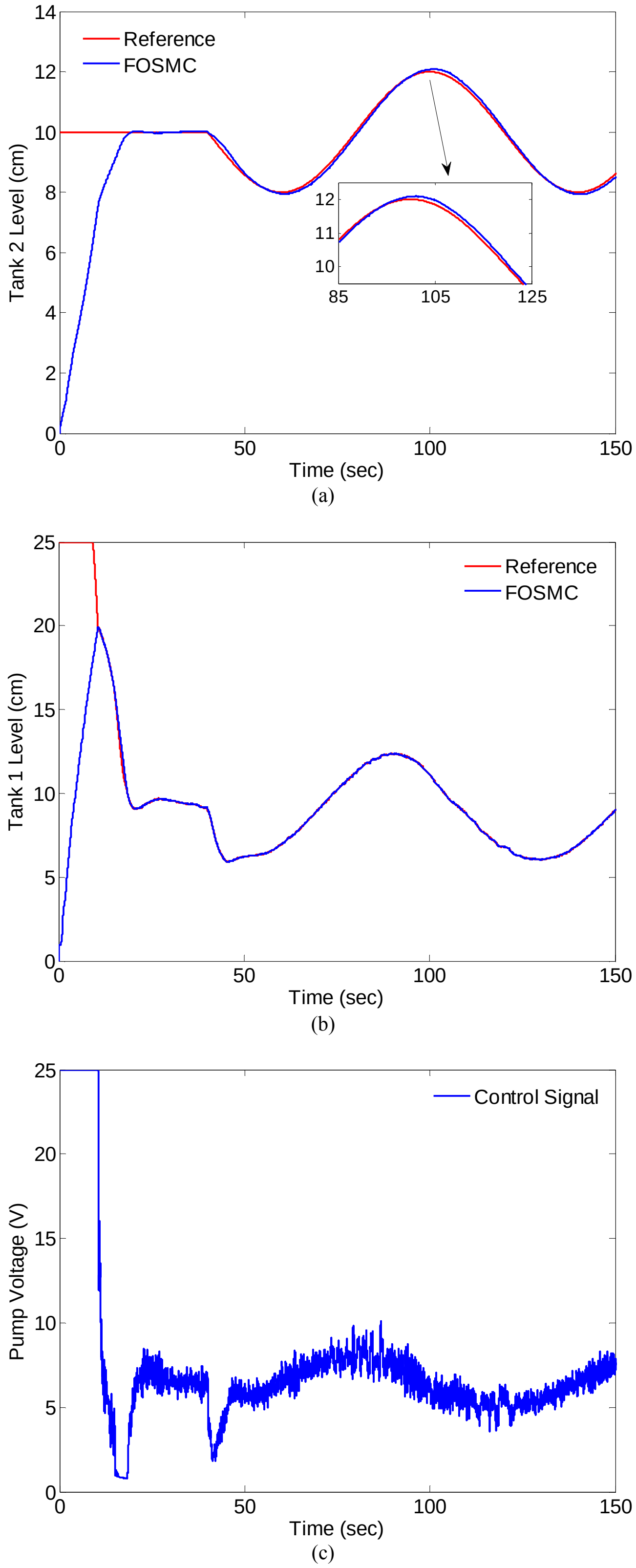


Figure 6. The FOSMC experimental results for configuration #2 under step + sinusoidal reference signal

In Figs. 5-6, the experimental results of configuration #2 are given for both controllers under step + sinusoidal reference signal. For configuration #2, the optimal parameter values for G and λ are determined as 85 and 10, respectively. Also, the fractional order derivative value r is taken as 0,5. From the figures, SMC has caught the step reference with greater rise and settling time. In addition, FOSMC has produced less wavy water reference level for bottom tank and tracked the sinusoidal reference signal with less deviation when it is compared with the result of SMC. Besides, FOSMC has produced more chattering control signal form to realize more sensitive control performance as well as having bigger control signal level as seen in the Figs. 5 (c) and 6 (c). Also, from the MAE results for tank 2, FOSMC has approximately 17% better trajectory tracking performance than SMC. FOSMC has also provided more flexible control performance to keep the system output at the desired water value.

TABLE I. MEAN ABSOLUTE ERROR OF CONTROLLERS' REFERENCE TRACKING

Controller	Reference Liquid Level of Tank 1	
	Step Plus Sawtooth	Step Plus Square
SMC	0.5317	0.6779
FOSMC	0.5121	0.6089
Total improvement	3.68%	10.17%

In addition, when the sinusoidal part of the reference signal is applied, FOSMC has shown better trajectory tracking performance with less deviation whereas SMC has some oscillations that causes undesired water adjustment control in the system between 105 to125 seconds.

TABLE II. MEAN ABSOLUTE ERROR OF CONTROLLERS' REFERENCE TRACKING

Controller	Reference Liquid Level of Tank 2
	Step Plus Sinusoidal
SMC	0.8309
FOSMC	0.6890
Total improvement	17.07%

V. CONCLUSION

In this paper, FOSMC is applied to a coupled tank system to control the water level of the upper tank and the bottom tank respectively. Also, to show the performance of the proposed controller, the SMC controller is applied in similar conditions to a coupled tank system. Experimental results show that FOSMC has improvements of 3.68 and 10.17% for upper tank and improvements of 17.07% for bottom tanks in terms of the reference tracking error when compared to SMC. The proposed control method also has lower rise and settling time in comparison to SMC. In addition, FOSMC has followed the time-varying reference signal with minimum deviation and has less trajectory tracking error level and is adequate to deal with the irregular water flow rate occurred during feeding of the bottom tank whereas it has generated the control signal form with more chattering.

REFERENCES

[1] S. Tunyasrirut, T. Suksri, A. Numsomran, S. Gulpanich and K. Tirasesth, "The Auto-Tuning PID controller for interacting water level process," Proceedings of World Academy of Science, Engineering and Technology, vol.1, no.12, pp.134-138, January 2007. doi.org/10.5281/zenodo.1072034
[2] K. Liu, "Advanced PID control and matlab simulation," Beijing: Publishing House of Electronics Industry, 2004.
[3] H. T. Sekban, K. Can and A. Başçi, "The Performance Analyze and Control of A Coupled Tank Liquid Level System by Fractional Order PI Controller," Turkish National Committee for Automatic Control (TOK), İstanbul, 2017, pp.126-131.
[4] H. T. Sekban, K. Can, and A. Başçi, "Real Time Application of Sliding Mode Controller for Coupled Tank Liquid Level System,"International Journal of Applied Mathematics, Electronics and Computers (IJAMEC), 2016, pp. 301-306.
[5] K. Can, H. T. Sekban and A. Başçi, "The Performance Analyze and Control of a Coupled Tank Liquid-Level System via PI & Backstepping Controllers," ELECO, Bursa, 2016, pp. 272-277.
[6] P. Boonsrimuang, A. Numsomran and S. Kangwanrat, "Design of PI controller using MRAC techniques for couple-tanks process," World Academy of Science Engineering and Technology, pp.67-72, 2009.
[7] A. K. Mahmood and H. H. Taha, "Design fuzzy logic controller for liquid level control," International Journal of Emerging Science and Engineering, 2013, pp. 24-26.
[8] A. Başçi and A. Derdiyok, "Implementation of an adaptive fuzzy compensator for coupled tank liquid level control," Measurement, vol.91, pp. 12-18., 2016. doi.10.1016/j.measurement.2016.05.026
[9] H. T. Sekban, "İkili tank sisteminde sıvı seviyesi kontrolünün kesir dereceli kayan kipli kontrolcü ile gerçekleştirilmesi," MSc thesis, Institute of science and Technology, Ataturk University, Erzurum, Turkey, 2017.
[10] K. C. Ng, Y. Li, D. J. Murray-Smith and K. C. Sharman, "Genetic algorithms applied to fuzzy sliding mode controller design," First international conference on genetic algorithms in engineering systems: innovations and applications (GALESIA), Sheffield, 1995, pp. 220-225. doi.10.1049/cp:19951052
[11] B. Moshiri, M. Jalili-Kharaajoo and F. Besharati, "Application of fuzzy sliding mode based on genetic algorithms to control of robotic manipulators," Emerging Technologies and Factory Automation, Lisbon, 2003, pp.169 – 172. doi: 10.1109/ETFA.2003.1248691
[12] R. Benayache, L. Chrifi-Alaoui, P. Bussy and J. M. Castelain, "Design and implementation of sliding mode controller with varying boundary layer for a coupled tanks system," 17th Mediterranean Conference on Cont. & Aut, 2009, pp. 1215-1220. doi.org/10.1109/MED.2009.5164712.
[13] N. B. Almutairi and M. Zribi, "Sliding mode control of coupled tanks," Mechatronics, vol.16, no.7, pp.427-441, September 2006. doi.org/10.1016/j.mechatronics.2006.03.001
[14] A. Levant, "Chattering Analysis," IEEE Transactions on Automatic Control, vol.55, no.6, pp.1380-1389, June 2010, doi. 10.1109/TAC.2010.2041973
[15] T. Floquet, S. K. Spurgeon and C. Edwards, "An Output feedback sliding mode control strategy for MIMO systems of arbitrary relative degree," International Journal of Robust and Nonlinear Control, 2011; pp. 119-133. doi. 10.1002/rnc.1579
[16] H. Abbas, S. Asghar and S. Qamar, "Sliding mode control of coupled tank liquid level control system," IEEE 10th International Conference on Frontires of Information Technology, Islamabad, 2012, pp. 325-330. doi. 10.1109/FIT.2012.65
[17] M. Ö. Efe and C. Kasnakoğlu, "A fractional adaptation law for sliding mode control," Int. J. Adapt. Control, 2008, pp.968-986. doi. 10.1002/acs.1062
[18] Quanser-Two Tank Manuel, 2005.
[19] R. Caponetto, G. Dongola, L. Fortuna and I. Petras, "Fractional order systems: modelling and control applications," World Scientific Series on Nonlinear Science Series A: 72; 2010.
[20] B. M. Vinagre, I. Podlubny, A. Hernandez and V. Feliu, "Some approximations of fractional order operators used in control theory and applications," Fractional Calculus and Applied Analysis 2000, pp.47-66.
[21] K. Orman, A. Basci, and A. Derdiyok, "Speed and Direction Angle Control of Four Wheel Drive Skid-Steered Mobile Robot by Using Fractional Order PI Controller, Elektronika Ir Elektrotechnika, vol.22, no.5, pp.14-19, 2016. doi.org/10.5755/j01.eie.22.5.16337
[22] I. Podlubny, "Fractional differential equations," New York: Academic Press, 1999.

[23] D. Valerio and J. S. Costa “Time domain implementation of fractional order controllers,” IET Proceedings-Control Theory and Applications, vol.152, no.5, pp.539-552, October 2005, doi.10.1049/ip-cta:20045063

[24] K. Orman, K. Can, A. Basci and A. Derdiyok, “An Adaptive-Fuzzy Fractional-Order Sliding Mode Controller Design for an Unmanned Vehicle”, Elektronika Ir Elektrotechnika, vol.24, no.2, pp.12-17, 2018. doi.org/10.5755/j01.eie.24.2.20630

[25] M. Aoun, R. Malti, F. Levron and A. Oustaloup A. “Nurnerical simulation of fractional systems,” ASME Design Engineering Technical Conference, Chicago, 2003, pp. 745-752. doi.org/10.1115/DETC2003/VIB-48389

[26] S. R. Mahapatro, “Control algorithms for a two tank liquid level system: An experimental study,” MSc thesis, National29 Institute of Technology, Odisha, India 2014.

[27] A. Derdiyok and A. Başçi, "The application of chattering-free sliding mode controller in coupled tank liquid-level control system," Korean Journal of Chemical Engineering, vol.30(3), pp.540-545, 2013. doi.org/10.1007/s11814-012-0177-y

[28] Eker İ. “Sliding mode control with PID sliding surface and experimental application to an electromechanical plant,” ISA Transactions, vol.45, no.1, pp.109-118. January 2006. doi.org/10.1016/S0019-0578(07)60070-6

[29] O. Özdal, “Model dayanaklı kayan kipli denetim,” MSc thesis, Hacettepe University, Ankara, Turkey, 2008

[30] Y. Longand and L. Li “Fuzzy fractional order sliding mode control for automatic clutch of vehicle AMT,” International Journal of Smart Home, vol.9, no.2, pp.53-68, January 2015, doi.10.14257/ijsh.2015.9.2.05

[31] V. I. Utkin, “Variable structure systems with sliding modes,” IEEE Transactions on Automatic Control, vol.22, no.2, pp. 212-222, April 1997. doi.10.1109/TAC.1977.1101446

[32] A. Basci, K. Can, K. Orman and A. Derdiyok, “Trajectory Tracking Control of a Four Rotor Unmanned Aerial Vehicle Based on Continuous Sliding Mode Controller”, Elektronika Ir Elektrotechnika, vol.23, no.3, pp.12-19, 2017. doi.org/10.5755/j01.eie.23.3.18325