Threshold Based Iteration Stopping Criterion for Turbo Codes and for Scheme Combining a Turbo Code and a Golden Space-Time Block Code

Alexandra SAVIN[1], Lucian TRIFINA[1], Mihaela ANDREI[2]
[1]*Gheorghe Asachi Technical University of Iaşi, 700506, Romania*
[2]*Dunărea de Jos University of Galaţi, 800146, Romania*
asavin@etti.tuiasi.ro, luciant@etti.tuiasi.ro, mihaela.andrei@ugal.ro

***Abstract*—This paper proposes an iteration stopping criterion for turbo decoding with Benedetto's decoding algorithm based on *a posteriori* probabilities. This stopping criterion is used in two schemes. Firstly, it is used in a classical turbo code scheme on additive white gaussian noise (AWGN) channel. Secondly, it is used in a scheme combining a turbo code and a Golden space-time block code on fast Rayleigh fading multiple input multiple output (MIMO) channel. Simulation results with different thresholds for the stopping criterion show that a threshold of 1.2 and 1.4 in the first and second scheme, respectively, are sufficient for obtaining the same bit error rate and frame error rate performance like in the case of using the ideal genie stopping criterion. The difference between the average number of iterations for these thresholds and for the genie stopping criterion is at most 1.5 and 1.25, respectively.**

***Index Terms*—average number of iterations, space-time block codes, stopping criterion, threshold, turbo codes.**

I. INTRODUCTION

A classical turbo code consists of the concatenation of two recursive systematic convolutional codes and an interleaver [1]. Turbo decoding can be done with algorithms based on sequence estimation (i.e. Soft-Output Viterbi Algorithm - SOVA) or algorithms based on symbol-by-symbol estimation (the soft input and soft output algorithm (SISO) Maximum *A Posteriori* (MAP)). The last ones have two versions: Bahl-Cocke-Jelinek-Raviv (BCJR) version [1] and SISO - *A Posteriori* Probability (APP) Benedetto version [2-3]. Of the two algorithm categories, the most frequently used are the ones based on symbol estimation. In practice, the BCJR algorithm uses the Log-MAP and Max-Log-MAP implementations, using log-likelihood ratios (LLRs) and the Jacobian logarithm approximation [4]. Benedetto's approach has the advantage of providing the soft estimates for the coded bits that can be used in schemes that combine a turbo code with another type of error correcting code, for example a space-time block code (STBC) [5-6].

Classical turbo decoding consists in performing a number of iterations between the two component decoders. The number of iterations can have a fixed value or can be determined by a specific iteration stopping criterion. Efficient iteration stopping criteria are very useful because they limit the decoding time without affecting the performances of bit error rates (BER) or frame error rates (FER) after decoding. So far, many iteration stopping criteria have been proposed for the BCJR-MAP decoding algorithm. A summarizing work is given in [7-8]. Stopping criteria for turbo codes are given in [9-20]. For the SISO–APP algorithm the literature doesn't provide us with specific results. However, iteration stopping criteria for BCJR-MAP can be implemented in the SISO-APP algorithm that works with probabilities. In this article, we propose the matching of the criterion based on the minimum absolute value of LLRs (minabsLLR) to SISO-APP Benedetto's decoding algorithm.

The paper is structured as follows: in section II the minabsLLR is revised for the BCJR-MAP decoding algorithm, in section III the proposed stopping criterion is presented (denoted minsumP), in section IV simulation results are shown for the classical turbo code scheme and the scheme combining a turbo code and a Golden STBC and section V concludes the paper.

This paper was realized with the support of POSDRU CUANTUMDOC "DOCTORAL STUDIES FOR EUROPEAN PERFORMANCES IN RESEARCH AND INNOVATION" ID79407 project funded by the European Social Fund and Romanian Government.

II. MINABSLLR STOPPING CRITERION FOR THE BCJR-MAP DECODING ALGORITHM

For the BCJR-MAP decoding algorithm the component decoders consist of the received values after demodulation corresponding to the BPSK modulated symbols: y_{sk} for the systematic bits, y_{p1k} and y_{p2k} for the parity check bits of the two component encoders, $k=1,2,...,N$, where N is the length of interleaver. The inputs in the first decoder are y_{sk} and y_{p1k} together with the *a priori* LLR values L_{ak} (initially set to 0 — for the first iteration $L_{ak}=0$, $k=1,2,...,N$). After the first half of iteration the extrinsic information sequence $L_{e1k}, k=1,2,...,N$, results. After interleaving, the *a priori* values $L_{a2k}, k=1,2,...,N$ of the second decoder result. This sequence is the third input of the second decoder together with the interleaved sequences y_s and y_{p2}. The output of the second decoder is represented by the extrinsic information sequence L_{e2}, that after interleaving becomes the *a priori* information of the first decoder L_{a1}. After a number of iterations (given or imposed by the stopping criterion) and after de-interleaving, the LLR_k values, are compared with the threshold 0 in order to provide the estimated bits.

Digital Object Identifier 10.4316/AECE.2014.01021

For the minabsLLR criterion [9-10] the iterations stop when all the LLR_k, $k = 1, 2, ..., N$ absolute values, at a given iteration, are greater or equal than the threshold value T_{LLR}:

$$\left|LLR_k\right| \geq T_{LLR}, \quad \forall k = 1, 2, ..., N \tag{1}$$

Otherwise, the iterations continue.

III. MinsumP Stopping Criterion for SISO-APP Decoding Algorithm

In this section, we propose the minabsLLR stopping criterion of the BCJR-MAP decoding algorithm, matched for the SISO-APP algorithm. This stopping criterion is denoted minsumP, because in this case the decision is taken by comparing sums of probabilities.

The algorithm is based on a SISO component decoder model, given in Figure 1 [2].

Figure 1. Soft-input soft-output model (SISO)

The SISO module is a four port device which has as inputs the probability distribution sequences

$$P(c;I) \;\; P(u;I) \tag{2}$$

and returns the probability distribution sequences

$$P(c;O) \;\; P(u;O) \tag{3}$$

based on the inputs and the component trellis code. I and O stand for input and output, respectively.

The algorithm is firstly presented for the two-step non-logarithmic version. The component code is a recursive convolutional systematic code with coding rate 1/2 (the uncoded bit is u, the systematic bit is c_s and the coded bit is c_p)

1) At time k the output probability distributions are computed as

$$P_k(c_p;O) = H_c \cdot \sum_{e:c(e)=c} A_{k-1}\left[s^S(e)\right] P_k\left[u(e);I\right] B_k\left[s^E(e)\right] \tag{4}$$

$$P_k(u;O) = H_u \cdot \sum_{e:u(e)=u} A_{k-1}\left[s^S(e)\right] P_k\left[c_p(e);I\right] B_k\left[s^E(e)\right] \tag{5}$$

2) $A_k(\cdot)$ and $B_k(\cdot)$ are obtained by forward and backward recursions, respectively:

$$A_k(s) = \sum_{e:s^E(e)=s} \left\{A_{k-1}\left[s^S(e)\right] P_k\left[u(e);I\right] P_k\left[c_s(e);I\right] P_k\left[c_p(e);I\right]\right\}, k = 1, \ldots, N \tag{6}$$

$$B_k(s) = \sum_{e:s^S(e)=s} \left\{B_{k+1}\left[s^E(e)\right] P_{k+1}\left[u(e);I\right] P_{k+1}\left[c_s(e);I\right] P_{k+1}\left[c_p(e);I\right]\right\}, k = N, \ldots, 1 \tag{7}$$

The initial values for the terminated trellis are:

$$A_0(s) = \begin{cases} 1, & s = S_0 \\ 0, & otherwise \end{cases} \tag{8}$$

$$B_N(s) = \begin{cases} 1, & s = S_0 \\ 0, & otherwise \end{cases} \tag{9}$$

For the unterminated trellis we have:

$$B_N(s) = 1/N_S, \, s = S_0, \ldots, S_{N_S-1}, \tag{10}$$

where N_s is the number of states in the code trellis. $s^S(e)$ and $s^E(e)$ in the above relations represent the initial and final state on branch e in the trellis section. $u(e)$ represents the systematic bit and $c(e)$ the coded bit from branch e.

H_c and H_u are normalization constants defined such that:

$$H_c \to \sum_{c_p} P_k(c_p;O) = 1 \tag{11}$$

$$H_u \to \sum_{u} P_k(u;O) = 1 \tag{12}$$

For the SISO-APP version of the turbo decoding algorithm from one decoder to the other we have the extrinsic values expressed by the probabilities $P_{ki}(u;O)$, where $i = 1$ for the first decoder and $i = 2$ for the second one. The systematic bit can take the values $u = 0$ or $u = 1$. The decisions are taken by comparing the value $P_{k1}(0;O) + P_{k2}(0;O)$ with $P_{k1}(1;O) + P_{k2}(1;O)$, $k = 1, \ldots, N$, providing to the output the bit corresponding to the greatest sum of probabilities.

Considering how the decision is made in the SISO-APP algorithm, we match the stopping criterion minsumP as follows: the iterations stop when all the values $P_{k1}(0;O) + P_{k2}(0;O)$, $k = 1, \ldots, N$, or all the values $P_{k1}(1;O) + P_{k2}(1;O)$, $k = 1, \ldots, N$, are greater than or equal to a certain threshold T_{sumP}, meaning:

$$P_{k1}(0;O) + P_{k2}(0;O) \geq T_{sumP}, \; \forall k = 1, \ldots, N \text{ or}$$
$$P_{k1}(1;O) + P_{k2}(1;O) \geq T_{sumP}, \forall k = 1, \ldots, N \tag{13}$$

Otherwise, the iterations will continue.

IV. Simulation Results and the Efficient Thresholds for the MinsumP Criterion

The simulation results will be given for the two schemes presented in the introduction: classical turbo code on AWGN channel (having the global coding rate of 1/3) and scheme combining a turbo code and a Golden STBC, like in [6]. In both cases, the generator matrix of component recursive systematic convolutional codes in the turbo code, in octal form, is $G = [1, 15/13]$. The interleaver used is a random interleaver of length 512. The used thresholds have a step of 0.2 and when the threshold is too low or too high (i. e. close to 1 or 2) the step is 0.1 or 0.05. When error rate performance is close to that for the genie stopping criterion, we use a step of 0.1 to improve the found threshold.

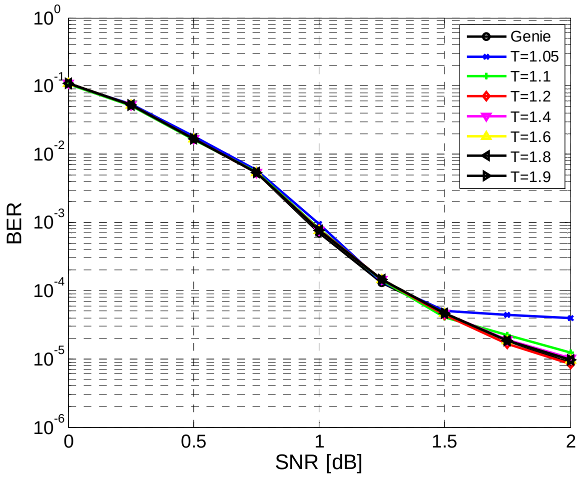
IV.1. Simulation Results for a Turbo Code on AWGN Channel

The simulation results for a classical turbo code on AWGN channel are given in Figure 2 a), the BER curves, b) the FER curves and in Figure 3 the curves for the average number of iterations of the ideal genie stopping criterion and the minsumP criterion for the following thresholds 1.05, 1.1, 1.2, 1.4, 1.6, 1.8 and 1.9, denoted with T.

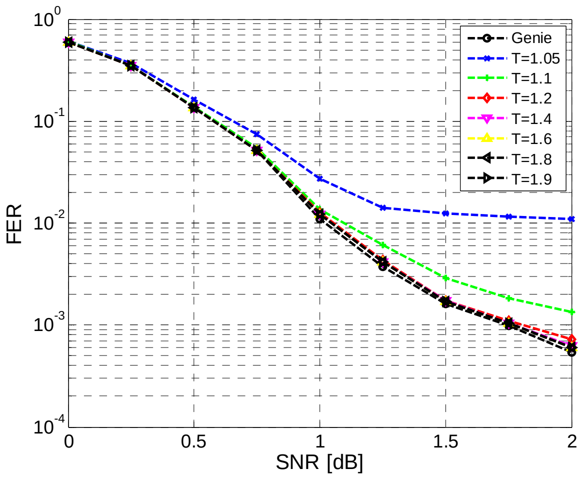
In Figure 2 a) using the genie stopping criterion we obtain similar BER values if the threshold is at least 1.1. In

Figure 2 b) we obtain similar FER values if the threshold is at least 1.2.

For the threshold of 1.2 the difference between the average number of iterations for the genie stopping criterion and the minsumP criterion is at most 1.5. At this threshold the average number of iterations for high SNR value (2 dB) is approximately 3, much lower than the maximum value of at least 8 recommended in literature.



a)



b)

Figure 2. a) BER and b) FER curves for the classical turbo code on AWGN channel

IV.2. Simulation Results for the Combining Scheme of a Turbo Code and a GOLDEN STBC

The simulation results for the scheme combining a turbo code and a Golden STBC on fast Rayleigh fading MIMO channel are given in Figure 4 a), the BER curves, Figure 4 b) the FER curves. Figure 5 represents the curves for the average number of iterations of the ideal genie stopping criterion and the minsumP criterion for the following thresholds 1.05, 1.1, 1.2, 1.3, 1.4, 1.6, 1.8 and 1.9, denoted with T.

In Figure 4 a) we obtain similar BER values with genie stopping criterion if the threshold is at least 1.3. In Figure 4 b) we obtain similar FER values if the threshold is at least 1.4.

For the threshold of 1.4 the difference between the average number of iterations for the genie stopping criterion and the minsumP criterion is at most 1.25, slightly lower than the corresponding value for the classical turbo code

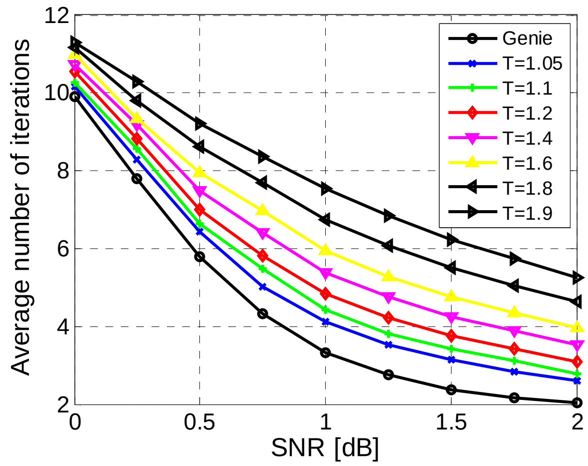
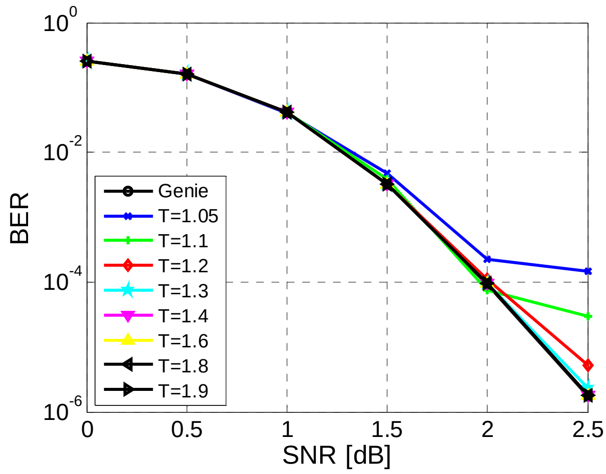
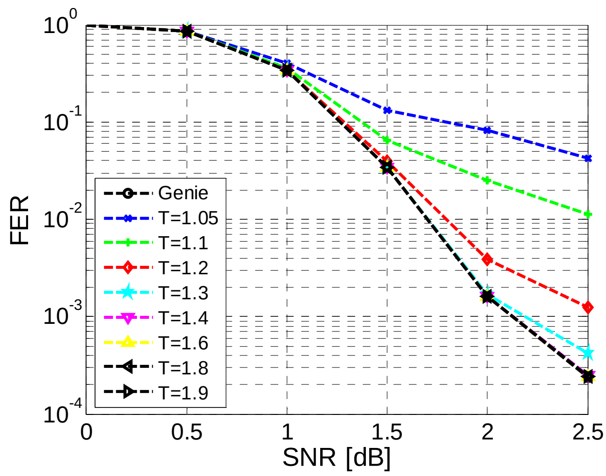


Figure 3. Average number of iterations for classical turbo code on AWGN channel



a)



b)

Figure 4. a) BER and b) FER curves for scheme combining a turbo code and a Golden STBC on fast Rayleigh fading MIMO channel

on AWGN channel. This fact can be due to the use of the Golden STBC which improves the soft value at input in the turbo decoder. At this threshold, the average number of iterations for high SNR value (2.5 dB) is approximately 3.

V. Conclusions

This paper proposes an iteration stopping criterion for turbo decoding using Benedetto's decoding algorithm based on *a posteriori* probabilities. The stopping criterion consists

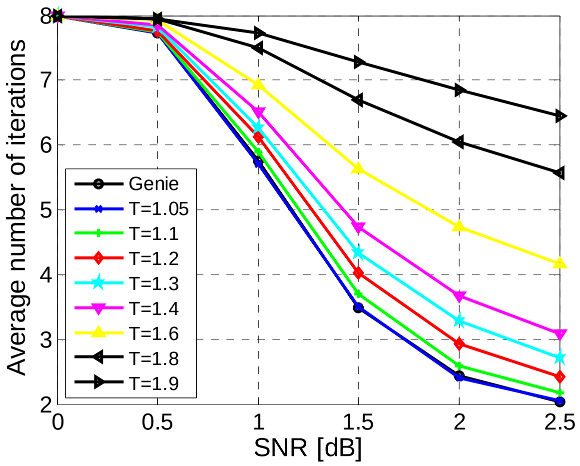


Figure 5. Average number of iterations for scheme combining a turbo code and a Golden STBC on fast Rayleigh fading MIMO channel

in stopping the decoding iterations when the sum of the two probabilities (of 0 and 1 after first and second component decoder, respectively) at a current time moment, resulted as extrinsic information, is greater than a given threshold for all bits in a frame.

This stopping criterion is used in a classical turbo code scheme on AWGN channel and in a scheme combining a turbo code and a Golden STBC on fast Rayleigh fading MIMO channel.

Simulation results with different thresholds of the stopping criterion from section 4 show that a threshold of 1.2 and 1.4 in the first and second scheme, respectively, are sufficient for obtaining the same BER and FER performances like in the case of using the ideal genie stopping criterion. The difference between the average number of iterations for these thresholds and the genie stopping criterion is at most 1.5 and 1.25, respectively.

REFERENCES

[1] C. Berrou, A. Glavieux, and P. Thitimajshima, “Near Shannon limit error correcting coding and decoding: Turbo-codes”, *IEEE Proceedings of International Conference on Communications ICC 1993,* Geneva, Switzerland, vol. 2, pp. 1064 –1070, 23-26 May 1993. [Online]. Available: http://dx.doi.org/10.1109/ICC.1993.397441

[2] S. Benedetto, D. Divsalar, G. Montorsi and F. Pollara, “A soft-input soft-output maximum a posteriori (MAP) module to decode parallel and serial concatenated codes”, *TDA Progress Report 42-127*, Nov. 1996

[3] S. Benedetto, D. Divsalar, G. Montorsy and F. Pollara, “A soft-input soft-output APP module for iterative decoding of concatenated codes”, *IEEE Communications Letters*, vol.1, no.1, pp. 22-24, Jan. 1997. [Online]. Available: http://dx.doi.org/10.1109/4234.552145

[4] P. Robertson, E. Villebrun and P. Hoeher, “A comparison of optimal and sub-optimal MAP decoding algorithms operating in the log domain”, *IEEE Proceedings of International Conference on Communications ICC 1995*, Seattle, Washington, pp. 1009-1013, June 1995. [Online]. Available: http://dx.doi.org/10.1109/ICC.1995.524253

[5] K. Amis, G. Sicot and D. Leroux, „Reduced complexity near-optimal iterative receiver for Wimax full-rate space-time code”, *5th International Symposium on Turbo Codes and Related Topics*, Lausanne, pp. 102-106, 1-5 Sept. 2008. [Online]. Available: http://dx.doi.org/10.1109/TURBOCODING.2008.4658680

[6] A. Savin and L. Trifina, ”Scheme combining a turbo code and a golden space-time block code with different interleavers”, *IEEE International Symposium on Signals, Circuits and Systems ISSCS 2013*, Iasi, Romania, pp. 37-40, 11-12 July 2013. [Online]. Available: http://dx.doi.org/10.1109/ISSCS.2013.6651189

[7] L. Trifina, D. Tărniceriu and H. Baltă, “Threshold determining for minabsLLR stopping criterion for turbo codes”, *Frequenz*, vol. 67, no. 9-10, pp. 321-326, Sept. 2013. [Online]. Available: http://dx.doi.org/10.1515/freq-2012-0159

[8] H. Baltă, C. Douillard and M. Kovaci, “The minimum likelihood APP based early stopping criterion for multi-binary turbo codes”, *Scientific Bulletin of “Politehnica” University from Timisoara*, vol. 51(65), no. 1-2, pp. 199-203, 21-22 Sept. 2006

[9] A. Matache, S. Dolinar and F. Pollara, “Stopping rules for turbo decoders,” *JPL TMO Progress Report*, vol. 42, pp.1–22, Aug. 2000

[10] L. Trifina, H.G. Baltă and A. Ruşinaru, “Decreasing of the turbo MAP decoding time by using an iterations stopping criterion”, *IEEE International Symposium on Signals, Circuits and Systems ISSCS 2005*, Iasi, Romania, pp. 371–374, 14-15 July 2005. [Online]. Available: http://dx.doi.org/10.1109/ISSCS.2005.1509933

[11] J. Hagenauer, E. Offer and L. Papke, “Iterative decoding of binary block and convolutional codes”, *IEEE Transactions on Information Theory*, vol. 42, no. 2, pp. 429–445, Mar. 1996. [Online]. Available: http://dx.doi.org/10.1109/18.485714

[12] R. Y. Shao, S. Lin and M. P. C. Fossorier, “Two simple stopping criteria for turbo decoding,” *IEEE Transactions on Communications*, vol. 47, no. 8, pp. 1117–1120, Aug. 1999. [Online]. Available: http://dx.doi.org/10.1109/26.780444

[13] Y. Wu, B. D. Woerner and W. J. Ebel, “A simple stopping criterion for turbo decoding”, *IEEE Communications Letters*, vol. 4, no. 8, pp. 258–260, Aug. 2000. [Online]. Available: http://dx.doi.org/10.1109/4234.864187

[14] T. M. N. Ngatched and F. Takawira, “Simple stopping criterion for turbo decoding”, *IEE Electronics Letters*, vol. 37, no. 22, pp. 1350-1351, Oct. 2001. [Online]. Available: http://dx.doi.org/10.1049/el:20010896

[15] A. Taffin, “Generalised stopping criterion for iterative decoders”, *IEE Electronics Letters*, vol. 39, no. 13, pp. 993-994, June 2003. [Online]. Available: http://dx.doi.org/10.1049/el:20030555

[16] A. Shibutani, H. Suda and F. Adachi, “Reducing average number of turbo decoding iterations,” *IEE Electronics Letters*, vol. 35, no. 9, pp. 701–702, Apr. 1999. [Online]. Available: http://dx.doi.org/10.1049/el:19990470

[17] Z. Ma, W. H. Mow and P. Fan, “On the complexity reduction of turbo decoding for wideband CDMA”, *IEEE Transactions on Wireless Communications*, vol. 4, no. 2, pp. 353-356, Mar. 2005. [Online]. Available: http://dx.doi.org/10.1109/TWC.2004.843050

[18] F. M. Li and A. Y. Wu, “On the new stopping criteria of iterative turbo decoding by using decoding threshold,” *IEEE Transactions on Signal Processing*, vol. 55, no. 11, pp. 5506–5516, Nov. 2007. [Online]. Available: http://dx.doi.org/10.1109/TSP.2007.899525

[19] L. Guerrieri, D. Veronesi and P. Bisaglia, “Stopping rules for duo-binary turbo codes and application to HomePlug AV”, *IEEE Global Telecommunications Conference GLOBECOM 2008*, pp. 2911-2915, 30 Nov. – 4 Dec. 2008. [Online]. Available: http://dx.doi.org/10.1109/GLOCOM.2008.ECP.558

[20] D. H. Kim and S. W. Kim, “Bit-level stopping of turbo decoding”, *IEEE Communications Letters*, vol. 10, no. 3, pp. 183-185, Mar. 2006. [Online]. Available: http://dx.doi.org/10.1109/LCOMM.2006.1603378