# FEM Analysis of Brushless DC Servomotor with Fractional Number of Slots per Pole 

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#### Abstract

The authors present in this paper the analysis with Finite Element Method (FEM) of the magnetic circuit for a Brushless DC servomotor with fractional number of slots/pole ( 9 slots and 10 poles). For this purpose, FEMM 4.2 software package was used for the analysis. To obtain the waveforms of Back-ElectroMotive Forces (BEMFs), electromagnetic and cogging torque for servomotor a program in LUA scripting language (integrated into interactive shell of FEMM4.2) has been created. A comparation with a structure with integer number of slots/pole ( 18 slots and 6 poles) was also realized. The analysis results prove that the structure chosen is an optimal solution: sinusoidal waveforms of BEMFs, improved electromagnetic torque and reduced cogging torque. Therefore, the operating characteristics of the servomotor with 9/10 slots/poles manufactured by Sistem Euroteh Company and included in an integrated electrical drives system are presented in this paper.


Index Terms-permanent magnet motors, brushless motors, finite element methods.

## I. Introduction

In most of the applications, in electrical drive systems, permanent magnet servomotors with high energy are used [3], [18]. In present, it is intended to replace classical DC servomotors by brushless servomotors either DC or AC. Therefore, in order to get high density of torque and power (torque and power versus total weight or total volume) special electrical machines having fractional number of slots per pole and phase, less than one, were designed [6], [17], [20]. The Fractional Number of Slots per Pole (FNS/P) is given by [1], [7]:

$$
\begin{equation*}
q=N_{S} /(2 \cdot m \cdot p) \tag{1}
\end{equation*}
$$

where: $N_{s}$ is the number of slots, $m$ is the number of phases and $p$ is the number of pole pairs.
Although the disadvantages related to the harmonic content of the stator magnetomotive force are presented, an electrical machine with FNS/P offers some advantages, such as [2], [6], [7], [8], [15], [19], [21]:
-higher specific output power and density torque;
-higher efficiency;
-lower cogging torque achieved;
-reduced winding mutual inductance between phases;
-reduced mass and volume of machine by lowering the stator copper content;
-high slot fill factor.
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To meet actual requirements, the designers of electrical machines have been tried to reduce the number of slots as much as possible with respect of a proper operation of the servomotor: trapezoidal or sinusoidal waveform for BEMFs and acceptable waveform and value for electromagnetic torque developed to the shaft (ripple and cogging torque reduced) [10].

In order to build a winding where a suitable sinusoidal voltage is induced, minimum 3 slots per pole and phase are required. For $2 \cdot p=10$ poles, minimum $N_{s}=2 \cdot 3 \cdot 5 \cdot 3=90$ slots are required [3]. According to the winding diagram from Fig. 1, there are 3 identical coils per phase, with a pitch of 110 , in serial connection. In these 3 coils, are induced BEMFs with same amplitude, but out of phase with an electrical angle $\alpha_{e}=180^{\circ} / 9=20^{\circ} \mathrm{el}$. If the middle coil is considered as a reference, the other two coils are shifted with $\pm 20^{\circ} \mathrm{el}$. (the sign depends on rotation direction of the machine). The waveform of resulting BEMFs on pole and phase is achieved through vector composition of the three BEMFs. For 10 poles, resulting BEMFs on pole can be with serial, parallel or mixed connection, with respect of winding configuration. The only solution to improve the BEMF waveform is to distribute the winding [11].


Figure 1. Partial winding 3-phase diagram of BLDCS with 90 slots and 10 poles (winding relative to a pole).

In the case of winding with 9 slots and 10 poles, according to the winding diagram from Fig. 2, there are 3 concentrated coils per phase, with a pitch of 1-2, in serial connection. In terms of execution, the winding is concentrated (a coil on stator tooth), but in terms of BEMFs, the winding is distributed in 3 identical coils for every phase. The geometric angle between coils is $360^{\circ} / 9=40^{\circ}$ and the geometric angle between poles is $360^{\circ} / 10=36^{\circ}$. If the middle coil is considered as a reference, the other two coils are shifted with a geometric angle of $\pm 4^{\circ}$ (the sign depends on rotation direction of the machine), so the electrical angle is $\pm 4 \cdot p= \pm 4 \cdot 5=20^{\circ} \mathrm{el}$., the same angle from classical winding from version with 90 slots.

From the above analysis, can be observed that in terms of performances, the winding with 9 slots and 10 poles is at least equivalent with the winding with 90 slots and 10 poles. In this context, the paper presents the magnetic circuit analysis based on Finite Element Method (FEM) of a BLDCS with FNS/P ( 9 slots and 10 poles). Finite Element Method Magnetics - FEMM 4.2 software package was used for this analysis. Therefore, this paper presents the operating characteristics of the servomotor with 9 slots and 10 poles realized at Sistem Euroteh Company and which was included in an Integrated Electrical Drives System (IEDS).


Figure 2. Winding 3-phase diagram of BLDCS with 9 slots and 10 poles.

## II. FEM ANALYsis

The final dimensions of servomotor's magnetic and electric circuit are achieved through numerical analysis of electromagnetic field based on FEM [9], [12-14], [16], [24]. For this purpose, it was used FEMM 4.2 software package that solves low frequency electromagnetic problems on twodimensional planar and axisymmetric domains. The program deals with linear or nonlinear magnetostatic and time harmonic magnetic problems, linear electrostatic problems, and steady-state heat flow problems [4].

For FEM analysis, a BLDCS with FNS/P (9 slots and 10 poles) was chosen. The main dimensions of this servomotor are presented in Fig. 3, and in Appendix A are presented two sequences from autocommutation process of this servomotor.


Figure 3. Cross-section of BLDCS with FNS/P: 1-PM, 2-stator stacking, 3-magnetic circuit, 4-air gap, 5-shaft, 6-consolidation magnets sheating.

The servomotor configuration in FEMM 4.2 was realized
based on results obtained by a simplified computation for dimensioning [23] (see Fig. 4). The main sizes of stator are computed in:

$$
\begin{gather*}
w_{t}=\frac{B_{m} \cdot b_{m}}{B_{S t} \cdot k_{\sigma S t}}=4.45 \mathrm{~mm}  \tag{2}\\
h_{y}=w_{t} / 2=2.225 \mathrm{~mm} \tag{3}
\end{gather*}
$$

where: $w_{t}$ is the tooth width, $h_{y}$ is the stator yoke height, $B_{m}$ is the magnetic flux density in central axys of permanent magnet, $b_{m}$ is the permanent magnet width, $B_{S t}$ is the average magnetic flux density in stator and $k_{\sigma S t}$ is the field dispersion coefficient between magnet and stator.


Figure 4. FEMM model of BLDCS with FNS/P (9 slots/10 poles-3651 nodes, 7156 elements).

The rotor magnetic circuit is realized from 430 Stainless Steel and for the 10 poles, NdFeB 35-SH permanent magnets were used. The stator stack was realized from M19 Steel. On each winding are 28 turns, the wire diameter being 0.5 mm . The shaft is made of the same material as the rotor magnetic circuit, which is why it is not represented in the model. The magnetic properties of the materials used in modeling are presented in Table I, Table II and the magnetization curves of these materials are shown in Fig. 5. The model resulted running the solver fkern.exe is presented in Fig. 6.

TABLE I. MAGNETIC PROPERTIES OF THE MATERIALS.

| Material | Electrical <br> Conductivity <br> $\sigma[\mathrm{MS} / \mathrm{m}]$ | Relative <br> Permeability <br> $\mu_{\mathrm{r}}$ | Fill <br> Factor <br> F | Thickness |
| :---: | :---: | :---: | :---: | :---: |
| $[\mathrm{mm}]$ |  |  |  |  |
| M-19 Steel | 1.9 | 4416 | 0.98 | 0.635 |
| Stainless <br> Steel 430 | 1.66 | 409 | 1 | 0.3 |

TABLE II. PERMANENT MAGNET USED IN ANALYSIS.

| Type | Remanent field density $\mathrm{B}_{\mathrm{r}}[\mathrm{T}]$ | Coercive force |  | Energy product $\mathrm{BH}_{\text {max }}$ $\left[\mathrm{kJ} / \mathrm{m}^{3}\right]$ |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{gathered} \mathrm{H}_{\mathrm{cb}} \\ {[\mathrm{kA} / \mathrm{m}]} \end{gathered}$ | $\begin{gathered} \mathrm{H}_{\mathrm{ci}} \\ {[\mathrm{kA} / \mathrm{m}]} \end{gathered}$ |  |
| NdFeB 35-SH | 1170-1220 | $\geq 876$ | $\geq 1592$ | 263-287 |



Figure 5. BH curves for M19 Steel and 430 Stainless Steel.


Figure 6. Flux distribution in the magnetic circuits of BLDCS with 9 slots and 10 poles.

Using LUA specific functions, the flux, electromagnetic torque (the difference between total torque and cogging torque), phase and line to line BEMFs were computed, keeping the speed constant. The waveforms of electromagnetic torque $T$, cogging torque $T_{c o g}$, phase BEMFs $\left(e_{A}, e_{B}, e_{C}\right)$ and line to line BEMFs $\left(e_{A B}, e_{B C}, e_{C A}\right)$ are shown in Fig. 7, Fig. 8, Fig. 9 and Fig. 10.


Figure 7. Waveforms of electromagnetic torque (9/10 slots/poles).
From the presented diagrams, one can see the maximum value of electromagnetic torque, 0.35 Nm , relatively low value of cogging torque (values between $\pm 0.002 \mathrm{Nm}$ ) and sinusoidal wavefors of BEMFs (phase and line to line).

In order to prove that the structure chosen is an optimal solution, this was compared with a structure with 18 slots
and 6 poles (Integer Number of Slots per Pole-INS/P) [22], presented in Fig. 11. In this situation, there are 6 coils on every phase in serial connection, with pitch of $1-4$. Permanent magnets are radially magnetized. Flux distribution in the magnetic circuits of BLDCS with 18 slots and 6 poles is shown in Fig. 12.


Figure 8. Waveforms of cogging torque (9/10 slots/poles).


Figure 9. Waveforms of phase BEMFs (9/10 slots/poles).


Figure 10. Waveforms of line to line BEMFs (9/10 slots/poles).
The results of this analysis are illustrated in Fig. 13, Fig. 14, Fig. 15 and Fig. 16. From diagrams presented in these figures, one can observ that in this situation BEMF waveform (phase and line to line) is not satisfactory. Furthermore, cogging torque has higher variations and values and electromagnetic torque ripple is much higher.

## III. Experimental Results

Research has been realized in the Electric Drives Laboratory of the Electrical Engineering Faculty, Gheorghe

Asachi Technical University of Iasi. The parameters of BLDCS with FNS/P (9/10 slots/poles) manufactured by Sistem Euroteh Company are presented in Tab. III and the general view is illustrated in Fig. 17. The servomotor manufactured is in star connection, equipped with position transducer with Hall sensors and it was included in an Integrated Electrical Drives System (IEDS) that also includes: resolver with no sliding contacts, speed/position incremental transducer with 4096 pulses/rev. and an electromagnetic brake (see Fig. 18). Therefore, a singlephase Synchronous Generator (SG) was manufactured to load the servomotor.


Figure 11. FEMM model of BLDCS with INS/P ( 18 slots/6 poles-14032 nodes, 27921 elements).


Figure 12. Flux distribution in the magnetic circuits of BLDCS with 18 slots and 6 poles.

As experimental results obtained through laboratory tests are presented:
-speed versus torque characteristic, Fig. 19;
-absorbed power versus absorbed current characteristic,
Fig. 20;
-efficiency versus absorbed current characteristic, Fig. 21;
-line to line BEMFs $e_{A B}$ and $e_{B C}$, Fig. 22.

## IV. Conclusions

In present, in most of the applications special electrical machines are used. The need to achieve high density of
power and torque contributed to the development of special machines with FNS/P.


Figure 13. Waveforms of electromagnetic torque (18/6 slots/poles).


Figure 14. Waveforms of cogging torque (18/6 slots/poles).


Figure 15. Waveforms of phase BEMFs (18/6 slots/poles).


Figure 16. Waveforms of line to line BEMFs (18/6 slots/poles).

TABLE III. RATED PARAMETERS OF THE BLDCS.

| Rated parameters | Symbol | Value | Unit |
| :---: | :---: | :---: | :---: |
| Number of phases | m | 3 | - |
| Number of poles | p | 10 | - |
| Rated Voltage | $\mathrm{U}_{\mathrm{n}}$ | 24 | V |
| Rated Current | $\mathrm{I}_{\mathrm{n}}$ | $3 . .5$ | A |
| Rated Speed | $\mathrm{n}_{\mathrm{n}}$ | 1725 | rpm |
| Rated Torque | $\mathrm{T}_{\mathrm{n}}$ | 0.35 | $\mathrm{~N} \cdot \mathrm{~m}$ |
| Rated Power | $\mathrm{P}_{\mathrm{n}}$ | 63.5 | W |
| L-L Resistance | R | 0.7 | $\Omega$ |
| L-L Inductance | L | 0.0013 | H |
| Voltage Constant | $\mathrm{K}_{\mathrm{e}}$ | 0.07 | $\mathrm{~V} /(\mathrm{rad} / \mathrm{s})$ |
| Rated Efficiency | $\eta_{\mathrm{n}}$ | 73 | $\%$ |



Figure 17. General view of the BLDCS with FNS/P manufactured by Sistem Euroteh: 1-shaft, 2-permanent magnet, 3-electromagnetic brake, 4bearing.


Figure 18. General view of the IEDS manufactured by Sistem Euroteh: 1BLDCS with FNS/P, electromagnetic brake, resolver, encoder, 2-elastic muff, 3-SG.


Figure 19. Speed versus torque characteristic.


Figure 20. Absorbed power versus absorbed current.


Figure 21. Efficiency versus absorbed current characteristic.


Figure 22. Line to line BEMFs $e_{A B}(\mathrm{CH} 1)$ and $e_{B C}(\mathrm{CH} 2)$.
To meet actual requirements, the designers of electrical machines have been tried to reduce the number of slots as much as possible with respect of a proper operation of the servomotor. Thereby, the paper presents FEM analysis of magnetic circuit of a BLDCS with FNS/P (9/10 slots/pole). For this purpose, FEMM 4.2 software package was used for the analysis. A comparation with a structure with integer number of slots per pole ( $18 / 6$ slots/pole) was also realized. The analysis results prove that the structure chosen ( $9 / 10$ slots/pole) is an optimal solution:
-sinusoidal waveforms of BEMFs;
-improved electromagnetic torque;
-reduced ripple electromagnetic torque;
-reduced cogging torque.
Therefore, this paper presents the operating (static) characteristics of the servomotor with $9 / 10$ slots per pole
manufactured by Sistem Euroteh Company and which was included in an IEDS.

a) $\left[0^{\circ} \div 12^{\circ}\right],(+\mathrm{A},-\mathrm{B})$

b) $\left[12^{\circ} \div 24^{\circ}\right],(-\mathrm{B},+\mathrm{C})$

Figure 23. Autocommutation of BLDCS with FNS/P ( 9 slots/10 poles).

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