

Codebook Generation Using Partition and Agglomerative Clustering

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Abstract—In this paper, we present a codebook generation algorithm to produce a codebook with lower distortion. Our method combines a fast codebook generation algorithm (CGAUCD) with doubling technique and fast agglomerative clustering algorithm (FACA) to generate a codebook with less computing time and lower distortion. Instead of using FACA directly to divide training vectors into M clusters, our proposed method first generates qM clusters from these training vectors, where $q > 1$ is an integer, and then applies FACA to merge these qM clusters into M cells. This is due to the computational complexity of CGAUCD with doubling technique is less than that of FACA. These M cluster centers are used as the initial codebook for CGAUCD. Using three real images as the training set, our method can reduce the MSE and computing time of FPNN+CGAUCD, which is the available best method to our knowledge, by 0.19 to 0.38 and 74.6% to 84.3%, respectively.

Index Terms—Codebook generation, agglomerative clustering, vector quantization

I. INTRODUCTION

Vector quantization (VQ) has been investigated in more than a decade [1-11] for data compression due to its relatively simple structure and computation. Many types of VQ, such as classified VQ [1- 4], finite state VQ [5], and side-match VQ [6], have been implemented for various purposes. VQ has been applied to some other applications including inverse halftoning [7-9]. The operations of VQ consist of separating the signal to be compressed into vectors (or blocks) and finding the closest codeword of an input vector from the codebook. This most similar codeword is called the reproduction vector of the input vector. In the encoding process of VQ, an index, which points to the closest codeword of an input vector, is found. In the decoding process, this index is used to fetch a codeword from the identical codebook.

A good codebook is important for VQ. The method most commonly used is the Linde-Buzo-Gary (LBG) algorithm [10, 11], which is also called the generalized Lloyd algorithm (GLA). GLA performs iteratively the partition step and new codebook generation step until convergence. The main drawback of GLA is that it gets stuck to the local optimal solution. To solve this problem, simulated annealing was presented [10]. However, simulated annealing requires a large amount of computing time and gains a little improvement only [12]. Another approach of obtaining a codebook is agglomerative clustering [13], which can usually obtain a better codebook than GLA [14]. For a data set of N training vectors, the computational complexities, in

terms of distance calculations, of GLA and agglomerative clustering are $O(NMt)$ and $O(N^3)$, respectively, where M is the number of codewords in a codebook, and t is the number of iterations. It is noted that $M \ll N$ and $t \ll N$, in general.

The generated codebook using GLA depends on the initial codebook. With a good initialization, a good codebook can be obtained by GLA. Using the cluster centers obtained by agglomerative clustering as the initialization for GLA, the corresponding codebook can further be improved. The computational complexities of GLA and agglomerative approach can be reduced by many available methods [14-16]. In [15], an FPNN (fast pairwise nearest neighbor) algorithm is proposed to reduce the computational complexity of agglomerative clustering. The computational complexity, in terms of the number of distance calculations, of FPNN is $O(\tau N^2)$, where τ is the average number of clusters to be updated for each merging process. In this paper, we will use the approaches presented in [16] to reduce the computational complexity of our proposed method. To use the developed fast agglomerative clustering algorithm, the nearest neighbor for each data points should be determined in the initialization process. To our knowledge, the fast k -nearest-neighbor search algorithm (FKNNSA) [17] is the fastest method and we will use it to determine the nearest neighbor of a data point for agglomerative clustering.

In this paper, we will use a method to divide a set of training vectors into qM clusters, where $q > 1$ is an integer and M is the codebook size. Then the developed fast agglomerative clustering algorithm is used to merge these qM clusters into M ones. Finally, CGAUCD (codebook generation algorithm using codeword displacement) [16] is used to generate the desired codebook of size M . It is noted that CGAUCD is a partitioning clustering algorithm.

This paper is organized as follows. Section II describes the related works. Section III presents the algorithms developed in this paper. Some experimental results are given in Section IV and concluding remarks are presented in Section V.

II. RELATED WORKS

In this section, we will describe CGAUCD (codebook generation algorithm using codeword displacement) [16], CGAUCD with doubling technique, and fast PNN algorithm [15].

(A) CGAUCD with Doubling Technique

The most well known algorithm for generating a

codebook is the generalized Lloyd algorithm (GLA). Denote the set of training vectors as $S = \{X\}$. Let $d(X, Y)$ be the distortion between any two vectors X and Y . In this paper, $d(X, Y)$ is defined as the squared Euclidean distance between X and Y . To reduce the computational complexity of GLA, we will use CGAUCD [16] to generate the desired codebook for it is a fast version of GLA and has the least computing time as far as we know.

The squared Euclidean distance between an input vector $X = (x_1, x_2, \dots, x_d)^t$ and a codeword $C = (c_1, c_2, \dots, c_d)^t$ is defined by the following equation:

$$d(X, C) = \left[\sum_{i=1}^d |x_i - c_i|^2 \right] \quad (1)$$

Let the center of cluster R_a in the current and previous partitions be C_a and C'_a , respectively. Denote the squared displacement between C_a and C'_a as D_a . If $D_a = 0$, then the vector C_a is defined as a static cluster center; otherwise it is called an active cluster center. If C_a is active, then R_a is called an active cluster; otherwise R_a is defined as a static cluster. Denote the squared Euclidean distances between a training vector X and the corresponding nearest as well as second nearest codewords in the previous partition as r'_1 and r'_2 , respectively.

Let the subcodebook CB_{active} consist of active codewords. Suppose that the training vector X is in a cluster R_a . Let $r_1 = d(X, C_a)$. In the case that R_a is static and a codeword C'_b is not the nearest codeword of X in the previous partition, then C'_b can't be the nearest codeword of X in the current partition also if C_b is static. If X is in an active cluster R_a , we will first calculate $r_1 = d(X, C_a)$. If $r_1 < r'_2$, the nearest codeword of X in the current iteration is impossible from the set of static codewords and we can find the closest codeword of X from CB_{active} . In the case of $r_1 \geq r'_2$, we must perform full search to find the closest codeword of X . Now, we would like to present CGAUCD below for the reason of completeness.

CGAUCD

- (1) Give an initial codebook CB_0 and preprocess all training vectors for fast search. Perform the partition process using codebook CB_0 , calculate r'_1 and r'_2 for each training vector X and generate a new codebook CB .
- (2) Generate the subcodebook CB_{active} and search structures for CB and CB_{active} .
- (3) For each training vector X , perform the following partition process:
 - 3a. If X is in a static cluster, search the codewords from CB_{active} to determine its nearest codeword and update r'_1 and r'_2 .
 - 3b. If X is in an active cluster with center C_a , calculate r_1 between X and C_a . If $r_1 < r'_2$, search subcodebook CB_{active} to find the nearest codeword of X and update r'_1 and r'_2 ; otherwise determine its nearest codeword of X from the codebook CB and update r'_1 and r'_2 .
- (4) Generate the new codebook CB and subcodebook

CB_{active} as well as the search structures for CB and CB_{active} .

- (5) Go to step 3 until the codebook is converged.

CGAUCD with doubling technique, referred to as CGAUCD+DT, generates M cluster centers in s steps, where $s = \log_2 M$. Let $SC(j) = \{C_1, C_2, \dots, C_{2^j}\}$ consist of 2^j cluster centers. To generate 2^{j+1} cluster centers using CGAUCD+DT, we first let $SC^*(j+1) = \{C^*_1, C^*_2, \dots, C^*_{2^{j+1}}\}$, where $C^*_i = C_{i/2} - \varepsilon$ if i is even; $C^*_i = C_{(i-1)/2} + \varepsilon$ if i is odd; and ε is a small random vector. It is noted that each component of ε is generated randomly to be a small positive real number. Using $SC^*(j+1)$ as the set of initial cluster centers, we can use CGAUCD to generate $SC(j+1) = \{C_1, C_2, \dots, C_{2^{j+1}}\}$. CGAUCD with doubling technique is presented as follows:

CGAUCD with Doubling Technique

- (1) Compute $SC(0) = \{C_1\}$ from the training set S , where $C_1 = \left(\sum_{i=1}^N X_i \right) / N$, and N is the number of data points. Set $s = \log_2 M$ and $j = 1$, where M is the number of clusters.
- (2) Let $SC^*(j) = \{C^*_1, C^*_2, \dots, C^*_{2^j}\}$, where $C^*_i = C_{i/2} - \varepsilon$ if i is even; $C^*_i = C_{(i-1)/2} + \varepsilon$ if i is odd. Use $SC^*(j)$ as the set of initial cluster centers for CGAUCD to generate $SC(j) = \{C_1, C_2, \dots, C_{2^j}\}$.
- (3) Set $j = j+1$. If $j > s$ stop; otherwise go to step (2).

(B)Fast PNN Algorithm

The fast PNN (pairwise nearest neighbor) algorithm, divides a set of N training vectors into M clusters through a sequence of merge operations. The increase of distortion of merging two clusters R_a and R_b into one cluster R_{ab} can be calculated by [18]

$$D_{a,b} = \frac{n_a n_b}{n_a + n_b} d(C_a, C_b) \quad (2)$$

where $n_a = |R_a|$; $n_b = |R_b|$; C_a is the center of R_a ; and C_b is the center of R_b . $D_{a,b}$ is called the cluster distance of R_a and R_b . The cluster center C_{ab} and cardinality n_{ab} (the number of training vectors) of R_{ab} are updated as follows:

$$C_{ab} = (n_a C_a + n_b C_b) / (n_a + n_b) \quad (3)$$

$$n_{ab} = n_a + n_b \quad (4)$$

At each stage of merge, two clusters which have the least cluster distance are determined and merged. For clarity, clusters R_a , R_b , and R_{ab} will be abbreviated to clusters a , b , and ab , respectively, in this paper. Fränti et al. [15] used a nearest neighbor table *NNT*, which records the index pointing to a cluster's nearest neighbor and the cluster

distance between the cluster and its nearest neighbor, to reduce the computational complexity of PNN algorithm. It is noted that $NNT[l]$ is the nearest neighbor for cluster l . If two clusters a and b are merged, then a set of clusters being updated (S_u) is determined, where $S_u = \{l: NNT[l] = a \text{ or } NNT[l] = b\} \setminus \{a, b\}$. Now, we would like to present the FPNN (fast PNN) algorithm.

FPNN Algorithm

- (1) For a cluster l ($l = 1, 2, \dots, N$), find the corresponding nearest neighbor $NNT[l]$ and generate the nearest neighbor table NNT . Set $N_t = N$.
- (2) Find two clusters b and a , which have the minimum cluster distance. Merge clusters a and b into cluster ab .
- (3) Find the set $S_u = \{l: NNT[l] = a \text{ or } NNT[l] = b\} \setminus \{a, b\}$.
- (4) Update C_{ab} and n_{ab} . Set $n_a = n_{ab}$, $C_a = C_{ab}$ and delete cluster b . For each cluster in S_u , find the corresponding nearest neighbor and update table NNT .
- (5) Set $N_t = N_t - 1$. If $N_t > M$, go to step (2).

III. PROPOSED ALGORITHM

To generate a codebook with lower distortion, we can use FPNN to generate a set of M cluster centers first. Then, these M cluster centers are used as the initial codebook for CGAUCD. To reduce the computational complexity of FPNN, we will present a fast agglomerative clustering algorithm in this section.

Let NN_l be the nearest neighbor of cluster l . For each cluster, we will also maintain a set of clusters, which have cluster l as their nearest neighbor. Denote $INNS_l$ as the set containing clusters, which have cluster l as their nearest neighbor. That is, $INNS_l = \{j: NN_j = l\}$. Denote cluster a as the nearest neighbor of cluster l . Here, we set $D_{min}(l) = D_{l,a}$. Let the cluster table CT record the cluster index and nearest distance table NDT store the corresponding nearest distance. It is noted that CT and NDT are arranged in the descending order of $D_{min}(l)$. From CT , we can easily find the clusters being merged. Denote these two clusters being merged as a and b . From $INNS_a$ and $INNS_b$, we can find the corresponding set of clusters whose nearest neighbors need to be updated. That is, the set of candidates being updated is $S_u = INNS_a \cup INNS_b \setminus \{a, b\}$. At this stage, we can calculate the nearest neighbor of cluster ab (NN_{ab}) and replace all the information of cluster a by the one of ab , where ab is the merged cluster.

In the beginning of determining the nearest neighbor for a cluster l , we first select a cluster and calculate its cluster distance to l . Let NN_l be the current nearest neighbor of l and denote r as the cluster distance between cluster l and NN_l . If another cluster j satisfies the follow expression:

$$D_{min}(j) \geq r \quad (5)$$

then cluster j can be rejected directly in the process of finding the closest neighbor for l . This is due to the cluster

distance between l and j should be greater than or equal to the nearest distance r . Since the cluster table CT is arranged in the descending order of cluster distance, we will have $NDT[j] \leq NDT[m]$ for $m < j$. That is, if expression (5) is satisfied by a cluster j and $CT[i] = j$, then $CT[i]$ won't be the nearest neighbor of l , where $t < i$. Using expression (5) to reject impossible candidates for a cluster's nearest neighbor is a novel approach.

We may expect that expression (5) can have a good performance of rejecting unlikely candidates for a cluster's nearest neighbor, if a good initial cluster is found. If clusters b and a have the minimum cluster distance we will merge them into cluster ab . To determine the nearest neighbor for cluster ab , we determine the initial candidate through finding the nearest neighbor NN_{ab} from the set $S_u = INNS_a \cup INNS_b \setminus \{a, b\}$. That is, NN_{ab} is used as the initial candidate for the nearest neighbor of cluster ab . In the case of $S_u = \emptyset$, if $CT[N_t-1] \neq b$ we set the initial nearest neighbor $NN_{ab} = CT[N_t-1]$; otherwise we let $NN_{ab} = CT[N_t-2]$, where \emptyset is the empty set and N_t is the number of clusters. To determine the initial nearest neighbor NN_l for a cluster $l \in S_u$, we can find it through finding the nearest neighbor of l from the set $INNS_l \setminus \{a, b\}$. In the case of $INNS_l \setminus \{a, b\} = \emptyset$, if $CT[N_t-1] \neq b$ we set the initial nearest candidate $NN_l = CT[N_t-1]$; otherwise let $NN_l = CT[N_t-2]$.

In the initialization step of our proposed fast agglomerative clustering algorithm, we need to determine the nearest neighbor for each data point. As far as we know, FKNNSA [17] is the fastest algorithm of determining a query point's nearest neighbor. Therefore, we will use it to determine NN_l for each cluster l in initialization step. FKNNSA uses a set of inequalities to reject impossible candidates for a query point during the process of finding its nearest neighbors. These inequalities are developed using the geometrical information of the query point and candidates. Now we would like to present our fast agglomerative clustering algorithm (FACA).

Fast Agglomerative Clustering Algorithm

- (1) Initially, allocate N data points to N clusters with each cluster having one data point.
 - (1a) For each cluster l ($l = 1, 2, \dots, N$), use FKNNSA to determine the corresponding nearest neighbor NN_l .
 - (1b) Generate the cluster table CT , nearest distance table NDT , and $INNS_l$.
 - (1c) Use the nearest distance table NDT to sort the cluster table CT by the descending order of cluster distance. Set $N_t = N$.
- (2) Find two clusters a and b , which have the minimum cluster distance from the cluster table CT , where $CT[N_t] = a$ and $NN_a = b$.
 - (2a) Merge clusters a and b into cluster ab and generate $S_u = INNS_a \cup INNS_b \setminus \{a, b\}$. Use equation (3) and (4) to update C_{ab} and n_{ab} . Set $n_a = n_{ab}$ and $C_a = C_{ab}$. Remove cluster b .
 - (2b) If $S_u \neq \emptyset$, determine NN_{ab} from the set S_u . In the case of $S_u = \emptyset$, if $CT[N_t-1] \neq b$ set $NN_{ab} = CT[N_t-1]$.

- 1]; otherwise set $NN_{ab} = CT[N_t - 2]$. Let $r = D_{ab,c}$, where $c = NN_{ab}$. Let $id_u = id = N_t - 1$.
- (2c) If $CT[id] = b$, set $id_b = id$ and go to step (2d). Calculate $D_{ab,c}$, where $c = CT[id]$. If $D_{ab,c} < r$, let $r = D_{ab,c}$ and set $NN_{ab} = CT[id]$. If $r < NDT[id]$, set $id_u = id$ and go to step (2e).
- (2d) Set $id = id - 1$ and go to step (2c).
- (2e) For $j = (id_u + 2)$ to id_b , set $CT[j] = CT[j-1]$ and $NDT[j] = NDT[j-1]$. Set $NN_a = NN_{ab}$, $CT[id_u + 1] = a$, and $NDT[id_u + 1] = r$. Let $m = NN_a$ and update $INNS_m = INNS_m \cup \{a\}$.
- (3) If S_u is an empty set, go to step (5); otherwise, for each cluster $l \in S_u$:
- (3a) If $INNS_l \setminus \{a, b\} \neq \emptyset$, set NN_l as the nearest neighbor of l determined from the set $INNS_l \setminus \{a, b\}$. In the case of $INNS_l \setminus \{a, b\} = \emptyset$, if $CT[N_t - 1] \neq b$ set $NN_l = CT[N_t - 1]$; otherwise let $NN_l = CT[N_t - 2]$. Let $r = D_{l,c}$, where $c = NN_l$. Let $id = id_u = N_t - 1$.
- (3b) If $CT[id] = l$, set $id_l = id$ and go to step (3c). Calculate $D_{ab,c}$, where $c = CT[id]$. If $D_{ab,c} < r$, let $r = D_{ab,c}$ and set $NN_l = CT[id]$. If $r < NDT[id]$, set $id_u = id$ and go to step (3d).
- (3c) Set $id = id - 1$ and go to step (3b).
- (3d) For $j = (id_u + 2)$ to id_b , set $CT[j] = CT[j-1]$ and $NDT[j] = NDT[j-1]$. Set $CT[id_u + 1] = l$ and $NDT[id_u + 1] = r$. Let $m = NN_l$ and update $INNS_m = INNS_m \cup \{l\}$.
- (3e) If cluster b is in $INNS_b$, update $INNS_l$ by removing cluster b from $INNS_l$.
- (4) Update $INNS_a$ from S_u . For each cluster c in S_u , insert cluster c into $INNS_a$ if NN_c is cluster a .
- (5) Set $N_t = N_t - 1$. If $N_t > M$, go to step (2).

The flow chart of FACA is given in figure 1. The computational complexity, in terms of the number of distance calculations, of FACA is $O(\tau\gamma N)$, where τ is the average number of clusters to be updated for each stage of cluster merge and γ is the average number of clusters to be visited in the process of determining a cluster's nearest neighbor. From our experiments, we find that the value of τ is about 4 and $\tau < N$.

The computational complexity of CGAUCD is $O(utN)$ [16], where N is the number of data points, $t < N$ is the number of iterations, and $u < M$ is the average number of clusters to be searched to determine a data point's nearest neighbor. CGAUCD with doubling technique (referred to as CGAUCD+DT) performs CGAUCD $\log_2 M$ times to generate a codebook of size M . Therefore the computational complexity of CGAUCD+DT is $O(utM \log_2 M)$. That is, the computational complexity of CGAUCD+DT is less than that of FACA. Instead of using FACA to merge N data points into M clusters directly, we can use CGAUCD+DT to generate a set of qM cluster centers first, where $q > 1$, and then apply FACA to merge these qM clusters into M cells to

reduce computing time. Finally, these M cluster centers will be used as the initial codebook for CGAUCD. This approach is referred to as algorithm 1 here. Now, we would like to present algorithm 1.

Algorithm 1

- (1) Use CGAUCD+DT to generate the set $SC = \{C_1, C_2, \dots, C_{qM}\}$, which consists of qM cluster centers.
- (2) Apply FACA to determine the initial codebook $CB = \{C_1, C_2, \dots, C_M\}$ with M codewords.
- (3) Use CGAUCD to generate the desired codebook.

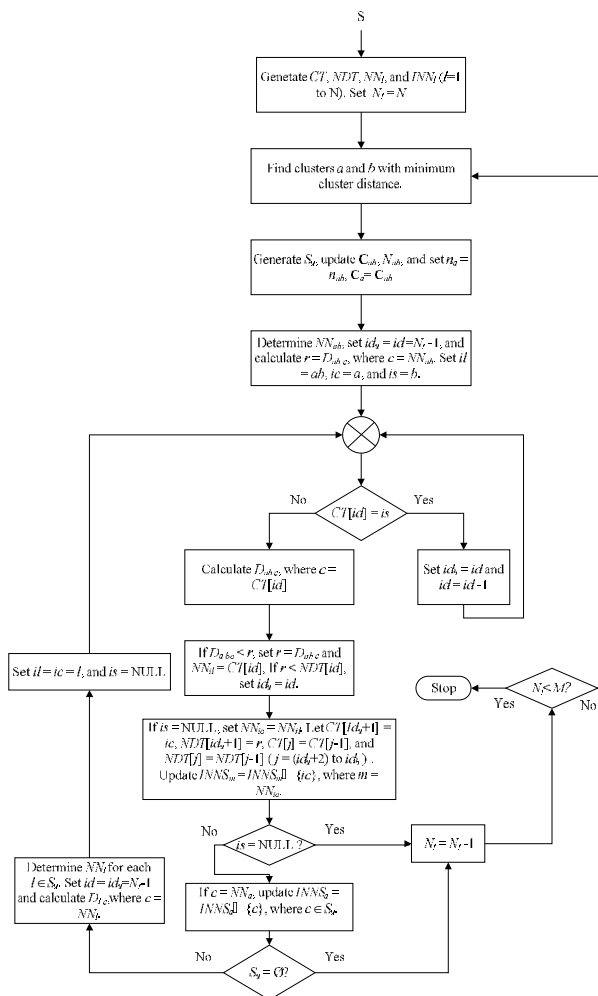
IV. EXPERIMENTAL RESULTS

To evaluate the performances of the proposed algorithms, three real data sets and several synthetic data sets have been used as our training sets to generate codebooks. The first real data set having 16,384 data points is obtained from a real image: "Lena," while the second real data set with 49,152 data points is generated from three real images: "Lena," "Baboon," and "Peppers." The third real data set with size of 98,304 is obtained from six real images: "Tiffany," "Peppers," "Baboon," "Airplane," "Bike" and "Girl." It is noted here that the characteristics of these images are different. The images "Baboon" and "Bike" possess many texture regions; whereas the pictures "Peppers" and "Tiffany" have a lot of smooth areas. "Girl" and "Airplane" have a small fraction of texture and edge areas. The synthetic data sets with size 10,000 and dimensions from 8 to 40 are obtained from the Gauss Markov sequence [10] with $\sigma=10$, $\mu=0$, and $\alpha=0.9$, where σ is the standard deviation, μ is the mean value of the sequence and α is the correlation coefficient. All computing is performed on a Pentium IV 3.2 GHz PC with 512 MB of memory.

Five codebook generation algorithms: GLA, FPNN, FPNN+CGAUCD, FACA+CGAUCD, and our method (algorithm 1) are implemented, where FPNN+CGAUCD and FACA+CGAUCD use the cluster centers generated by FPNN and FACA, respectively, as the initial codebooks for CGAUCD. For the GLA, the initial codewords are selected randomly from the training set. These methods are compared in terms of computing time and mean square error (MSE). The mean square error (MSE) for a set of N training vectors $\{X_i\}$ is defined as follows:

$$MSE = \left(\sum_{i=1}^N \|X_i - Q(X_i)\|^2 \right) / (Nd) \quad (6)$$

where $Q(X_i)$ is the closest codeword (reproduction vector) of X and d is the vector dimension. MSE is usually used to measure the quality of a picture [10]. The lower MSE implies the better image quality. That is, a codebook with lower MSE implies that it is a better codebook. Therefore MSE is used here as a metric to measure the quality of a codebook.



runs, FPNN, FPNN+CGAUCD, FACA+CGAUCD and algorithm 1 for the second data set.

TABLE 6: THE LEAST MEAN SQUARE ERROR OF GLA WITH 100 RUNS AS WELL AS MEAN SQUARE ERRORS OF PNN, PNN+CGAUCD, FACA+CGAUCD AND ALGORITHM 1 USING TRAINING VECTORS FROM THE THIRD REAL DATA SET

Method	M		
	128	256	512
GLA with 100 runs	153.946964	129.462589	109.456045
FPNN	158.015196	132.853107	112.043991
FPNN+CGAUCD	153.419479	128.515097	108.244302
FACA+CGAUCD	153.033692	128.805776	108.185836
Algorithm 1, $q=2$	153.737790	128.575970	108.116461
Algorithm 1, $q=4$	153.252590	128.612478	108.057259

TABLE 7: THE COMPUTING TIME OF GLA WITH 100 RUNS, PNN, PNN+CGAUCD, FACA+CGAUCD AND ALGORITHM 1 USING TRAINING VECTORS FROM THE THIRD REAL DATA SET

Method	M		
	128	256	512
GLA with 100 runs	273.53	436.03	909.18
FPNN	6817.52	6779.39	6737.73
FPNN+CGAUCD	6882.86	6863.53	6840.41
FACA+CGAUCD	2018.39	1685.56	1636.61
Algorithm 1, $q=2$	321.47	452.91	583.51
Algorithm 1, $q=4$	433.97	564.84	777.80

From table 2 and table 4, we will find that FPNN gives highest mean square error in average and our method (algorithm 1 with $q = 4$) gives the least mean square errors in almost all cases. Compared to GLA, algorithm 1 with $q = 4$ can reduce the mean square error by 0.69 to 5.40. Tables 2 and 4 show that both FPNN+CGAUCD and FACA+CGAUCD can obtain about the same mean square error for a codebook. It is noted that there is a little difference between the means square errors of the codebooks generated by FPNN+CGAUCD and FACA+CGAUCD, respectively. In the early stage of merging clusters, there are many cluster pairs with the same cluster distance. Merging different pairs with the same cluster distance in the early stage will result in a little difference for the clustering results. That is, there will be a little difference between the means square errors of the codebooks generated by FPNN+CGAUCD and FACA+CGAUCD due to FPNN and FACA may select different cluster pairs with the same cluster distance. Our method FACA+CGAUCD can reduce the computing time of FPNN+CGAUCD by about 32.9% for the second real data set. From table 3, we will also find that the computing time of algorithm1 with $q = 2$ or 4 is less than that of FACA+CGAUCD. This is due to the computational complexity of CGAUCD+DT is less than that of FACA. It is noted that to our knowledge, FPNN+CGAUCD is the available algorithm of generating a codebook with the lowest mean square error. Compared to FPNN+CGAUCD, algorithm 1 with $q = 2$ can generate a codebook with the much less computing time and little higher MSE. Compared

with FPNN+CGAUCD, algorithm 1 with $q = 4$ generates a codebook with the much less computing time and lower MSE. Compared to FPNN+CGACD, algorithm 1 with $q = 4$ can reduce the average mean square error and computing time by 0.165 and 79.4%, respectively. From tables 3 and 4, we can find that the MSE of algorithms 1 is lower in general when a larger q is used. However, it should be noted that the corresponding computing time is also increased when a larger q is adopted. From table 2, we can conclude that algorithm 1 with $q = 4$ is not global optimal. Therefore algorithm 1 with $q = 2$ may occasionally obtain lower MSE than algorithm 1 with $q = 4$ (for example, $M = 128$), although their difference is not significant any more.

Figures 2 to 4 presents the compressed images of “Lena,” “Baboon,” and “Peppers,” using codebooks of size 256 generated by GLA with 100 runs, FPNN, FPNN+CGAUCD, and algorithm 1 with $q=2$, respectively. From figures 2(a), 2(b), 2(c), and 2(d), we can find that GLA and FPNN have more blocking effects in the hat; while FPNN+CGAUCD and algorithm 1 provide less blocking effects and better image quality. For figures 3(a) to 4(d), these four algorithms have almost the same visual quality.

To understand the effect of data size on our proposed methods, a large data set is used here. Table 6 presents the least MSE of GLA with 100 runs as well as MSEs of FPNN, FPNN+CGAUCD, FACA+CGAUCD and algorithm 1 for the third real data set; while table 7 shows the computing time of GLA with 100 runs, FPNN, FPNN+CGAUCD, FACA+CGAUCD and algorithm 1 for the same data set. From table 7, we can find that our method algorithm 1 with $q = 2$ has the least computing time in average. Compared to FPNN+CGAUCD, algorithm 1 with $q = 2$ can reduce the computing time significantly and obtain a little higher MSE. From table 2 to table 7, we can conclude that the performances of our proposed methods are more remarkable when a larger data set is used.

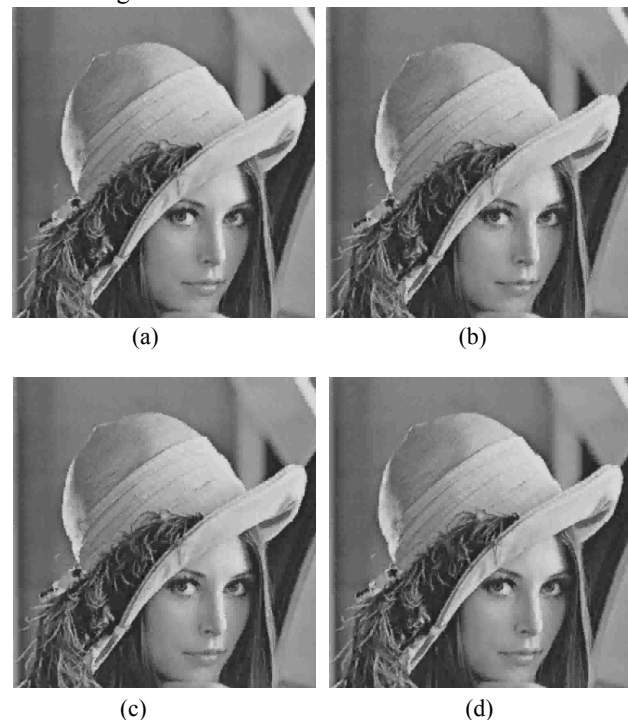


Figure 2. The compressed images of “Lena” using codebooks generated by (a) GLA with 100 runs; (b) FPNN, (c) FPNN+CGAUCD, and (d) algorithm 1 with $q=2$.

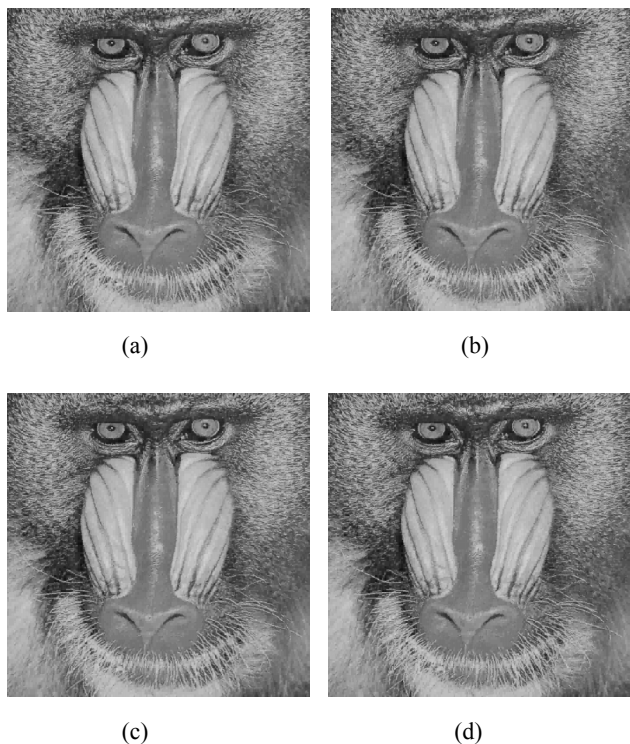


Figure 3. The compressed images of “Baboon” using codebooks generated by (a) GLA with 100 runs; (b) FPNN, (c) FPNN+CGAUCD, and (d) algorithm 1 with $q=2$.

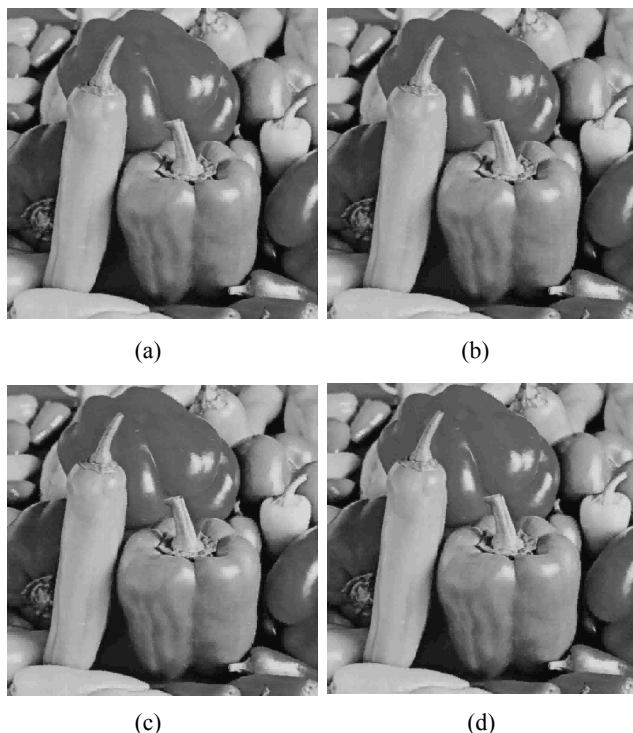


Figure 4. The compressed images of “Peppers” using codebooks generated by (a) GLA with 100 runs; (b) FPNN, (c) FPNN+CGAUCD, and (d) algorithm 1 with $q=2$.

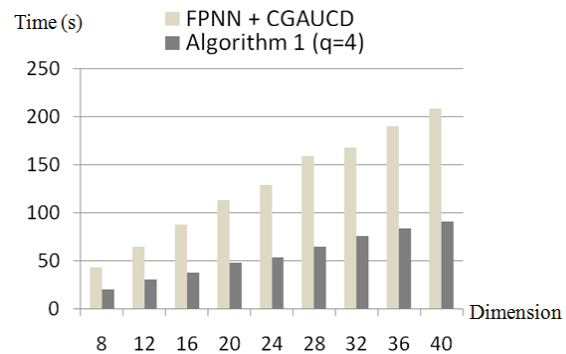


Figure 5. The computing time (in seconds) of generating codebooks of size 256 using synthetic data sets with sizes = 10,000 and dimensions from 8 to 40.

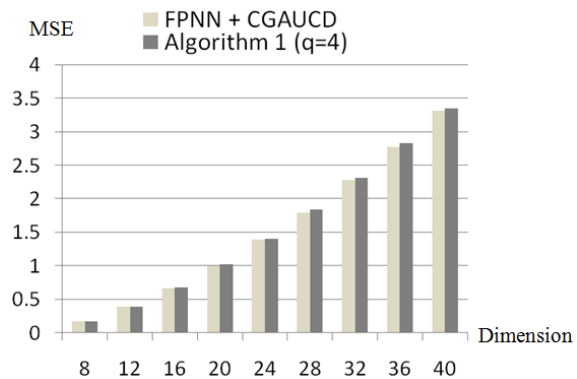


Figure 6. The mean square errors for synthetic data sets of generating codebooks of size 256 using synthetic data sets with sizes = 10,000 and dimensions from 8 to 40.

To study the effect of data dimension on the performances of our proposed methods, figures 5 and 6 present the mean square errors and computing time, respectively, of FPNN+CGACD and algorithm 1 with $q = 4$ to generate codebooks of size 256 using synthetic data sets with size = 10,000 and dimension ranging from 8 to 40. From figures 4 and 5, we can find that compared to FPNN+CGAUCD, our method algorithm 1 with $q = 4$ can reduce the average computing time of 60.8% with about the same MSE. The performance of algorithm 1 is better when a data set with higher dimension is used. It is recommended that algorithm 1 with $q = 4$ can be used to generate the desired codebook for it gives the least mean square error.

V. CONCLUSION

In this paper, we develop a method to generate a codebook from a set of training vectors. A fast agglomerative clustering algorithm FACA is also developed to reduce the computing time of FPNN. FACA uses an inequality to speed up the process of finding a cluster's nearest neighbor. Compared to the available best method as far as we know, our method algorithm 1 with $q = 4$ can reduce the average mean square error by 0.165 with the reduction of computing time by 82.3% to 89.3% for the second real data set. Compared to GLA, our method algorithm 1 with $q = 4$ can decrease the mean square error of a codebook by 0.69 to 5.40. Compared to FPNN+CGAUCD, which is the available best method to our knowledge, our proposed method FACA+CGAUCD can decrease the computing time by about 32.9% with the same mean square error for the second real data set.

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