

Small Speed Asymptotic Stability Study of an Induction Motor Sensorless Speed Control System with Extended Gopinath Observer

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Abstract—The paper presents a synthesis of an extended Gopinath observer (EGO) and analyzes the asymptotic stability of a squirrel-cage induction motor vector control system with an EGO in its loop. The considered control system is based on the direct rotor flux orientation method (DFOC) and the study of stability is based upon the linearization theorem applied around the equilibrium points of the control system, emphasizing the estimated variation domain of the rotor resistance for which the control system remains asymptotically stable.

Index Terms—extended Gopinath observer, induction motor, vector control system, direct rotor flux orientation method, asymptotic stability, sensorless.

I. INTRODUCTION

This paper proposes a new flux and rotor speed observer called Extended Gopinath observer (EGO). The design of the EGO observer was conceived based on an adaptive mechanism using the notion of Popov hyperstability [1].

Thus, this type of observer is included in the estimation methods based on an adaptation mechanism, along with the Extended Luenberger [2] Observer (ELO) proposed by Kubota [3] and the Model Adaptive System (MRAS) observer proposed by Schauder [4].

In the second part of the paper a study about the asymptotic stability of the whole speed control system which contains in his loop an EGO observer has been performed.

II. THE EXTENDED GOPINATH OBSERVER

The EGO observer is composed of a low order Gopinath rotor flux observer [5],[6] and an adaptation mechanism used for the rotor speed estimation. The equations that define the rotor flux Gopinath observer are [7]:

$$\begin{cases} \frac{d}{dt} \hat{i}_s = a_a^* \cdot \hat{i}_s + a_b^* \cdot \hat{i}_s + a_{12}^* \cdot \hat{\psi}_r + b_{11}^* \cdot u_s \\ \frac{d}{dt} \hat{\psi}_r = a_{21}^* \cdot \hat{i}_s + a_{22}^* \cdot \hat{\psi}_r + g \cdot \left[\frac{d}{dt} \hat{i}_s - \frac{d}{dt} \hat{i}_s \right] \end{cases} \quad (1)$$

where: $T_s^* = \frac{L_s^*}{R_s^*}; T_r^* = \frac{L_r^*}{R_r^*};$

$$a_{11}^* = a_a^* + a_b^*; a_{12}^* = a_{13}^* - j \cdot a_{14}^* \cdot z_p \cdot \hat{\omega}_r; a_{21}^* = a_{31}^*;$$

$$a_{13}^* = \frac{L_m^*}{L_s^* \cdot L_r^* \cdot T_r^* \cdot \sigma^*}; a_{14}^* = \frac{L_m^*}{L_s^* \cdot L_r^* \cdot \sigma^*}; a_{31}^* = \frac{L_m^*}{T_r^*};$$

$$a_{22}^* = a_{33}^* + j \cdot z_p \cdot \hat{\omega}_r; a_{33}^* = -\frac{1}{T_r^*}; b_{11}^* = \frac{1}{L_s^* \cdot \sigma^*};$$

$$a_a^* = -\frac{1}{T_s^* \cdot \sigma^*}; a_b^* = -\frac{1 - \sigma^*}{T_r^* \cdot \sigma^*}; \sigma^* = 1 - \frac{L_m^{*2}}{L_s^* \cdot L_r^*}.$$

In the above relations we marked with “*” the identified electrical sizes of the induction motor.

The block diagram of the EGO is presented in Figure 1.

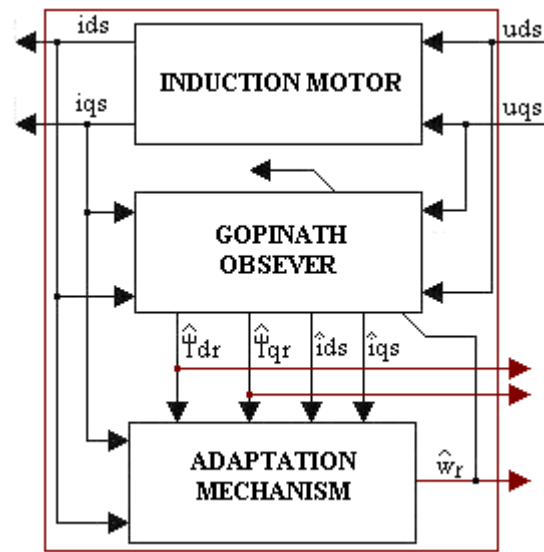


Figure 1. The Principle Schematic of the EGO Estimator

The essential element that determines the flux observer's stability, as well as its lack of sensibility to the motor parameters variation, g is a matrix Gopinath, represented by a complex number of the form: $g = g_a + j \cdot g_b$.

In order to design the estimator, its poles have to be placed in the left complex plane. In this case, the stability of the estimator is assured. The expressions g_a and g_b after the pole positioning become [7]:

$$\begin{cases} g_a = -k \cdot \frac{a_{31}^* \cdot a_{33}^*}{(a_{33}^*)^2 + (z_p \cdot \hat{\omega}_r)^2} \\ g_b = k \cdot \frac{a_{31}^* \cdot z_p \cdot \hat{\omega}_r}{(a_{33}^*)^2 + (z_p \cdot \hat{\omega}_r)^2} \end{cases}; k > 0 \quad (2)$$

In these conditions the Gopinath rotor flux observer is

completely determined. Next, in order to determine the adaptation mechanism to estimate the rotor speed, as reference model the „statoric currents - rotoric fluxes” one of the induction motor and as adjustable model, the model of the Gopinath rotor flux observer have been considered.

The equations mentioned above, written under the input-state-output canonic form become:

- Reference model:

$$\begin{cases} \frac{d}{dt}x = A \cdot x + B \cdot u \\ y = C \cdot \frac{d}{dt}x \end{cases} \quad (3)$$

- Adjustable model:

$$\begin{cases} \frac{d}{dt}\hat{x} = \tilde{A} \cdot \hat{x} + A_1 \cdot x + B \cdot u + \tilde{G} \cdot (y - \hat{y}) \\ \hat{y} = C \cdot \frac{d}{dt}\hat{x} \end{cases} \quad (4)$$

where:

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}; \tilde{A} = \begin{bmatrix} a_a^* & a_{12}^* \\ 0 & a_{22}^* \end{bmatrix}; A_1 = \begin{bmatrix} a_b^* & 0 \\ a_{21}^* & 0 \end{bmatrix}$$

$$\tilde{G} = \begin{bmatrix} 0 \\ g \end{bmatrix}; x = \begin{bmatrix} i_s \\ \psi_r \end{bmatrix}; \hat{x} = \begin{bmatrix} \hat{i}_s \\ \hat{\psi}_r \end{bmatrix}; u = \underline{u}_s; B = \begin{bmatrix} b_{11} \\ 0 \end{bmatrix}.$$

In the above relations we marked with „ \sim ” the matrices of the Gopinath estimator which are dependent upon the rotor speed, which in turn need to be estimated based on the adaptation mechanism.

Next, in order to determine the expression that defines the adaptation mechanism it has to be assumed that the identified electric sizes are identical with the real electric sizes of the induction motor. In other words:

$$a_{ij} = a_{ij}^*; i, j = 1, 2 \text{ and } b_{11} = b_{11}^* \quad (5)$$

First, in order to build the adaptive mechanism, the estimation error is evaluated, defined by the following difference:

$$e_x = x - \hat{x} \quad (6)$$

By deriving the relation (6) in relation with time and by using the relations (3) and (4), the relation (6) becomes:

$$\frac{d}{dt}e_x = (A - A_1) \cdot x - \tilde{A} \cdot \hat{x} - \tilde{G} \cdot C \cdot \frac{d}{dt}e_x \quad (7)$$

If the determinant $\det(I_2 + \tilde{G} \cdot C) \neq 0$ then exists a unique inverse matrix $M = (I_2 + \tilde{G} \cdot C)^{-1}$ and hence the expression (7) could be written as follows:

$$\frac{d}{dt}e_x = M \cdot (A - A_1) \cdot e_x + M \cdot (A - A_1 - \tilde{A}) \cdot \hat{x} \quad (8)$$

Equation (8) describes a linear system defined by the term $M \cdot (A - A_1) \cdot e_x$ in inverse connection with a non linear system defined by the term $\Phi(e_y)$ that receives as input the error $e_y = C \cdot e_x$ between the models and gives as output the following term:

$$\rho = -M \cdot (A - A_1 - \tilde{A}) \cdot \hat{x} \quad (9)$$

The block diagram of the system that describes the

dynamic evolution of the error between the state of the reference model and the state of the adjustable model is presented in Figure 2:

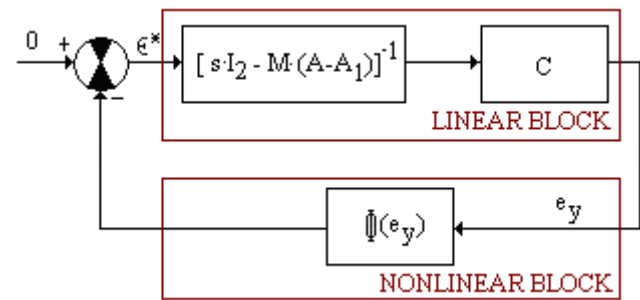


Figure 2. Lur'e problem block diagram

As one may notice, this problem is frequently mentioned by the non-linear systems literature, being exactly the configuration of the Lur'e problem, and of one of the problems treated by Popov [1].

Considering, according to the Popov terminology, the non-linear block described by, $\Phi(e_y)$ the integral input-output index associated, is the following:

$$\eta(0, t_1) = \text{Re} \left(\int_0^{t_1} e_y^T(t) \cdot \rho(t) dt \right) \quad (10)$$

The necessary hyper-stability condition for the above mentioned block is given by:

$$\eta(0, t_1) \geq -\gamma^2(0) \quad (11)$$

for any input-output combination and where $\gamma(0)$ is a positive constant.

In the above relation we marked with e_y^T the following expression:

$$e_y^T = \begin{bmatrix} \bar{e}_y & 0 \end{bmatrix} \quad (12)$$

in order to keep the compatibility between the input and output dimensions; \bar{e}_y represents the conjugate of the complex variable e_y .

Under these circumstances, using the relation (9), expression (11) becomes:

$$-\text{Re} \left(\int_0^{t_1} e_y^T \cdot M \cdot (A - A_1 - \tilde{A}) \cdot \hat{x} dt \right) \geq -\gamma^2(0) \quad (13)$$

Next we assume that the error $M \cdot (A - A_1 - \tilde{A})$ is determined only by the rotor speed of the induction motor.

In this case one may write:

$$M \cdot (A - A_1 - \tilde{A}) = (\omega_r - \hat{\omega}_r) \cdot A_{er} \quad (14)$$

where:

$$A_{er} = \begin{bmatrix} 0 & -j \cdot a_{14} \cdot z_p \\ 0 & j \cdot z_p \cdot (1 + a_{14} \cdot g) \end{bmatrix}$$

ω_r and $\hat{\omega}_r$ is real and estimated mechanical angular speed.

For any positive derivable f function the following inequality is true:

$$K_1 \cdot \int_0^{t_1} \left(\frac{df}{dt} \cdot f \right) dt \geq -\frac{K_1}{2} \cdot f^2(0) \quad (15)$$

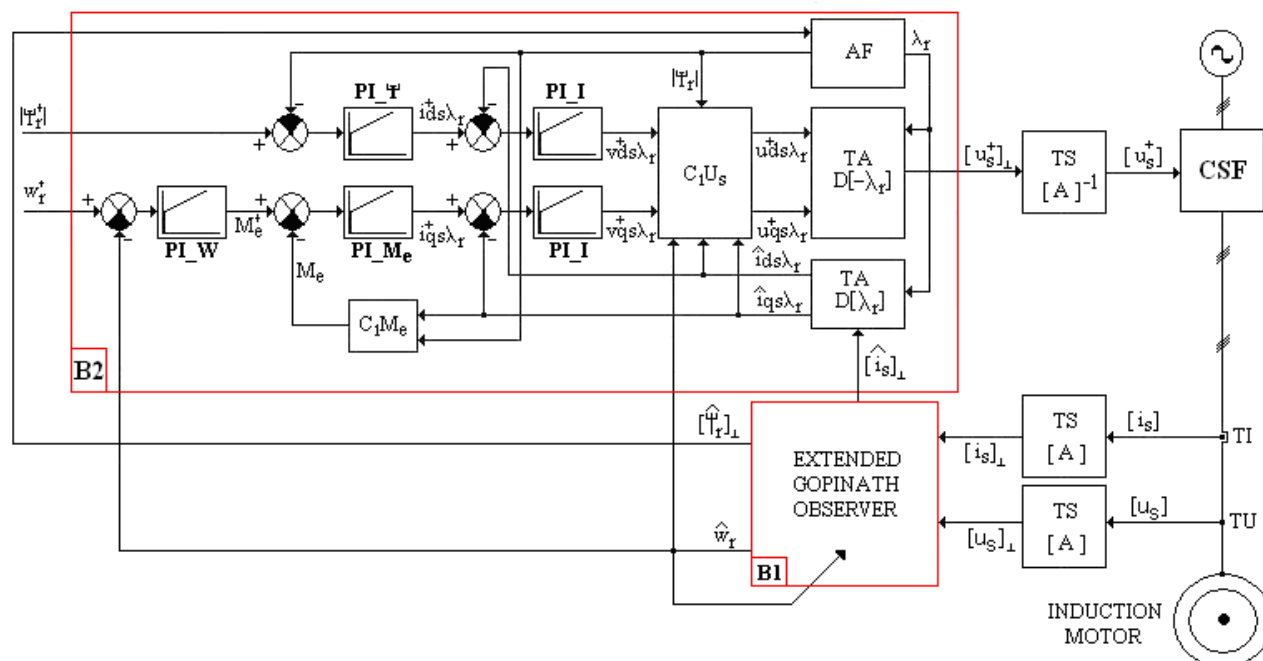


Figure 3. The block diagram of the DFOC vectorial control system which contains an EGO loop

On the other hand, using the relation (14), the expression (13) becomes:

$$-\operatorname{Re}\left(\int_0^{t_1} e_y^T \cdot A_{er} \cdot \hat{x} \cdot (\omega_r - \hat{\omega}_r) dt\right) \geq -\gamma^2(0) \quad (16)$$

By combining the relations (15) and (16) one may write the following relations:

$$\begin{cases} f = \omega_r - \hat{\omega}_r \\ -\text{Re}(e_y^T \cdot A_{er} \cdot \hat{x}) = K_1 \cdot \frac{df}{dt} \end{cases} \quad (17)$$

Because K_1 is a constant and then, in case of a slower ω_r parameter variation related to the adaptive law, we get:

$$\hat{\omega}_r = k_i \cdot \int_0^{t_1} \text{Re}(e_y^T \cdot A_{er} \cdot \hat{x}) dt \quad (18)$$

After replacing the variables that define the above expression (18) and taking into account the arbitrary nature of the k_i positive constant we get:

$$\hat{\omega}_r = k_i \cdot \int_0^{t_1} (e_{yd} \cdot \hat{\psi}_{qr} - e_{yq} \cdot \hat{\psi}_{dr}) dt \quad (19)$$

where: $e_{vd} = i_{ds} - \hat{i}_{ds}$ and $e_{vq} = i_{qs} - \hat{i}_{qs}$.

Sometimes, insted of the adaptation law (19) the following form could be used:

$$\hat{\omega}_r = K_R(e_{yd}\hat{\psi}_{qr} - e_{yq}\hat{\psi}_{dr}) + K_i \int_0^{t_1} (e_{yd}\hat{\psi}_{qr} - e_{yq}\hat{\psi}_{dr}) dt \quad (20)$$

where: $K_i = K_R / T_R$

From the above relation one can observe that a new proportional component appears from the desire of having 2 coefficients that could control the speed estimation dynamics. This fact isn't always necessary because very good results by using only expression (19) could be obtained.

Thus, expression (20) represents the general formula of the adaptation mechanism where K_R represents the proportionality constant and T_R represents the integration time of the proportional-integral controller that defines the adaptation mechanism.

III. THE MATHEMATICAL DESCRIPTION OF THE VECTOR CONTROL SYSTEM

The block diagram of the direct rotor flux oriented control system of the mechanical angular speed ω_r (DFOC) is presented in Figure 3.

In Figure 3, B2 is the control block of the rotor flux oriented speed control (DFOC) and with B1 the extended Gopinath observer blocks (EGO).

In order to mathematically describe the DFOC control system, the following hypotheses have been considered:

- The static frequency converter (CSF) is assumed to contain a voltage inverter.
- The static frequency converter is considered ideal, so that the vector of the command is considered to be the input vector of the induction motor.
- The transducers are considered to be ideal.
- The mathematical model of the vectorial control system is written in an orthogonal axis system $d\lambda_r - q\lambda_r$ bound to the rotor flux module.

Some of the equations that define the vector control system are given by the elements which compose the field orientation block and consist of:

- stator voltage decoupling block ($C_1 U_s$):

$$\begin{cases} u_{ds\lambda_r}^* = \frac{1}{b_{11}^*} \cdot [b_{11}^* \cdot v_{ds\lambda_r}^* - h_1] \\ u_{qs\lambda_r}^* = \frac{1}{b_{11}^*} \cdot [b_{11}^* \cdot v_{qs\lambda_r}^* + h_2] \end{cases} \quad (21)$$

where:

$$h_1 = a_{13}^* \cdot |\psi_r| + a_{31}^* \cdot \frac{\hat{i}_{qs\lambda_r}^2}{|\psi_r|} + z_p \cdot \hat{\omega}_r \cdot \hat{i}_{qs\lambda_r}$$

$$h_2 = a_{14}^* \cdot z_p \cdot \hat{\omega}_r \cdot |\psi_r| + a_{31}^* \cdot \frac{\hat{i}_{ds\lambda_r} \cdot \hat{i}_{qs\lambda_r}}{|\psi_r|} + z_p \cdot \hat{\omega}_r \cdot \hat{i}_{ds\lambda_r}$$

- PI flux controller (PI_ψ) defined by the $K_ψ$ proportionality constant and the $T_ψ$ integration time:

$$\begin{cases} \frac{dx_6}{dt} = \psi_r^* - |\psi_r| \\ i_{ds\lambda_r}^* = \frac{K_ψ}{T_ψ} \cdot x_6 + K_ψ \cdot (\psi_r^* - |\psi_r|) \end{cases} \quad (22)$$

- torque PI controller (PI_{M_e}) defined by the K_M proportionality constant and the T_M integration time:

$$\begin{cases} \frac{dx_7}{dt} = M_e^* - M_e \\ i_{qs\lambda_r}^* = \frac{K_M}{T_M} \cdot x_7 + K_M \cdot (M_e^* - M_e) \end{cases} \quad (23)$$

- mechanical angular speed PI controller (PI_ω) defined by the $K_ω$ proportionality constant and the $T_ω$ integration time:

$$\begin{cases} \frac{dx_8}{dt} = \omega_r^* - \hat{\omega}_r \\ M_e^* = \frac{K_ω}{T_ω} \cdot x_8 + K_ω \cdot (\omega_r^* - \hat{\omega}_r) \end{cases} \quad (24)$$

- current PI controller (PI_I) defined by the K_i proportionality constant and the T_i integration time:

$$\begin{cases} \frac{dx_9}{dt} = \hat{i}_{ds\lambda_r}^* - \hat{i}_{ds\lambda_r} \\ v_{ds\lambda_r}^* = \frac{K_i}{T_i} \cdot x_9 + K_i \cdot (\hat{i}_{ds\lambda_r}^* - \hat{i}_{ds\lambda_r}) \end{cases} \quad (25)$$

$$\begin{cases} \frac{dx_{10}}{dt} = \hat{i}_{qs\lambda_r}^* - \hat{i}_{qs\lambda_r} \\ v_{qs\lambda_r}^* = \frac{K_i}{T_i} \cdot x_{10} + K_i \cdot (\hat{i}_{qs\lambda_r}^* - \hat{i}_{qs\lambda_r}) \end{cases} \quad (26)$$

- Flux analyser (AF):

$$|\psi_r| = \sqrt{\hat{\psi}_{dr}^2 + \hat{\psi}_{qr}^2}; \sin \lambda_r = \frac{\hat{\psi}_{qr}}{|\psi_r|}; \cos \lambda_r = \frac{\hat{\psi}_{dr}}{|\psi_r|} \quad (27)$$

- The calculus of the torque block (C₁M_e):

$$M_e = K_a \cdot |\psi_r| \cdot \hat{i}_{qs\lambda_r}; K_a = 3 \cdot z_p \cdot L_m^* / (2 \cdot L_r^*) \quad (28)$$

The other equations that define the mathematical model of the speed vector control system are:

- The induction motor: The equations that define the stator currents – rotor currents mathematical model of the induction motor, is [8]:

$$\frac{d}{dt} \begin{bmatrix} i_s \\ i_r \end{bmatrix} = \begin{bmatrix} \alpha_a & \alpha_b \\ \alpha_c & \alpha_d \end{bmatrix} \cdot \begin{bmatrix} i_s \\ i_r \end{bmatrix} + \begin{bmatrix} \beta_{11} \\ \beta_{31} \end{bmatrix} \cdot u_s \quad (29)$$

$$\text{where: } \alpha_a = \alpha_{11} - j \cdot \alpha_{12} \cdot \omega; \alpha_b = \alpha_{13} - j \cdot \alpha_{14} \cdot \omega; \\ \alpha_c = \alpha_{31} + j \cdot \alpha_{32} \cdot \omega; \alpha_d = \alpha_{33} + j \cdot \alpha_{34} \cdot \omega;$$

$$i_r = i_{dr} + j \cdot i_{qr}; \alpha_{11} = -\frac{1}{T_s \cdot \sigma}; \alpha_{12} = \frac{1 - \sigma}{\sigma}; \alpha_{13} = \frac{L_m}{L_s \cdot T_r \cdot \sigma}$$

$$\alpha_{14} = \frac{L_m}{L_s \cdot \sigma}; \alpha_{31} = \frac{L_m}{L_r \cdot T_s \cdot \sigma}; \alpha_{32} = \frac{L_m}{L_r \cdot \sigma}; \omega = z_p \cdot \omega_r;$$

$$\alpha_{33} = -\frac{1}{T_r \cdot \sigma}; \alpha_{34} = \frac{1}{\sigma}; \beta_{11} = \frac{1}{\sigma \cdot L_s}; \beta_{31} = -\frac{L_m}{L_s \cdot L_r \cdot \sigma}$$

The motion equation of the induction motor proper to the stator currents – rotor fluxes model, is:

$$\frac{d}{dt} \omega_r = K_{m1} \cdot (i_{dr} \cdot i_{qs} - i_{qr} \cdot i_{ds}) - K_{m2} \cdot \omega_r - K_{m3} \cdot M_r \quad (30)$$

where: $K_{m1} = 3 \cdot z_p \cdot L_m / (2 \cdot J)$; $K_{m2} = F / J$; $K_{m3} = 1 / J$.

The equations that define the extended Gopinath observer defined by the 4 relations that can be written based on system (1) with the equation that defines the speed adaptation mechanism (20). Expression (20) can also be written as:

$$\begin{cases} \frac{dx_{15}}{dt} = (i_{ds} - \hat{i}_{ds}) \cdot \hat{\psi}_{qr} - (i_{qs} - \hat{i}_{qs}) \cdot \hat{\psi}_{dr} \\ \hat{\omega}_r = \frac{K_R}{T_R} \cdot x_{15} + K_R \cdot [(i_{ds} - \hat{i}_{ds}) \cdot \hat{\psi}_{qr} - (i_{qs} - \hat{i}_{qs}) \cdot \hat{\psi}_{dr}] \end{cases} \quad (31)$$

All these expressions form a 14 differential equations system with 14 unknown values. In order to offer a coherent presentation of this differential equations system, we have used the following notations:

- The state vector of the control system will be:

$$x = [x_i]_{i=1,14} \quad (32)$$

where: $x_1 = i_{ds\lambda_r}$; $x_2 = i_{qs\lambda_r}$; $x_3 = i_{dr\lambda_r}$; $x_4 = i_{qr\lambda_r}$; $x_5 = \omega_r$;

$$x_{11} = \hat{i}_{ds\lambda_r}; x_{12} = \hat{i}_{qs\lambda_r}; x_{13} = \hat{\psi}_{dr\lambda_r}$$

- The input vector of the control system is:

$$u = [u_1 \quad u_2 \quad u_3]^T \quad (33)$$

where: $u_1 = \omega_r^*$; $u_2 = \psi_r^*$; $u_3 = M_r$.

Under these circumstances the 14 differential equations system that define the mathematical model of the vector control system can be written as follows

$$\dot{x} = f(x, u) \quad (34)$$

where: $f(x, u) = [f_i(x, u)]_{i=1,14}$

$$f_1 = \alpha_{11} \cdot x_1 + \alpha_{12}^a \cdot x_2 + \alpha_{13} \cdot x_3 + \alpha_{14} \cdot z_p \cdot x_5 \cdot x_4 + \beta_{11} \cdot g_1 \quad (35)$$

$$f_2 = -\alpha_{12}^a \cdot x_1 + \alpha_{11} \cdot x_2 - \alpha_{14} \cdot z_p \cdot x_5 \cdot x_3 + \alpha_{13} \cdot x_4 + \beta_{11} \cdot g_2 \quad (36)$$

$$f_3 = \alpha_{31} \cdot x_1 - \alpha_{32} \cdot z_p \cdot x_5 \cdot x_2 + \alpha_{33} \cdot x_3 + \alpha_{34}^a \cdot x_4 + \beta_{31} g_1 \quad (37)$$

$$f_4 = \alpha_{32} \cdot z_p \cdot x_5 \cdot x_1 + \alpha_{31} \cdot x_2 - \alpha_{34}^a \cdot x_3 + \alpha_{33} \cdot x_4 + \beta_{31} g_2 \quad (38)$$

$$f_5 = K_{m1} \cdot (x_3 \cdot x_2 - x_4 \cdot x_1) - K_{m2} \cdot x_5 - K_{m3} \cdot u_3 \quad (39)$$

$$f_6 = u_2 - x_{13} \quad (40)$$

$$f_7 = \frac{K_ω}{T_ω} \cdot x_8 + K_ω \cdot (u_1 - g_3) - K_a \cdot x_{13} \cdot x_{12} \quad (41)$$

$$f_8 = u_1 - g_3 \quad (42)$$

$$f_9 = \frac{K_ψ}{T_ψ} \cdot x_6 + K_ψ \cdot (u_2 - x_{13}) - x_{11} \quad (43)$$

$$f_{10} = \frac{K_M}{T_M} \cdot x_7 + K_M \cdot f_7 - x_{12} \quad (44)$$

$$f_{11} = a_a^* \cdot x_{11} + \hat{\omega}_{\lambda_r} \cdot x_{12} + a_b^* \cdot x_1 + a_{13}^* \cdot x_{13} + b_{11}^* \cdot g_1 \quad (45)$$

$$f_{12} = -\hat{\omega}_{\lambda_r} x_{11} + a_a^* x_{12} + a_b^* x_2 - a_{14}^* z_p g_3 \cdot x_{13} + b_{11}^* g_2 \quad (46)$$

$$f_{13} = a_{31}^* \cdot x_1 + a_{33}^* \cdot x_{13} + g_a (f_1 - f_{11}) - g_b (f_2 - f_{12}) - g_b \cdot \hat{\omega}_{\lambda_r} \cdot (x_1 - x_{11}) - g_a \cdot \hat{\omega}_{\lambda_r} \cdot (x_2 - x_{12}) \quad (47)$$

$$f_{14} = -x_{13} \cdot (x_2 - x_{12}) \quad (48)$$

where:

$$v_{ds} = \frac{K_i}{T_i} \cdot x_9 + K_i \cdot f_9; v_{qs} = \frac{K_i}{T_i} \cdot x_{10} + K_i \cdot f_{10} \quad (49)$$

$$h_1 = a_{13}^* \cdot x_{13} + a_{31}^* \cdot \frac{x_{12}^2}{x_{13}} + z_p \cdot g_3 \cdot x_{12} \quad (50)$$

$$h_2 = a_{14}^* \cdot z_p \cdot g_3 \cdot x_{13} + a_{31}^* \cdot \frac{x_{11} \cdot x_{12}}{x_{13}} + z_p \cdot g_3 \cdot x_{11} \quad (51)$$

$$g_1 = \frac{b_{11}^* \cdot v_{ds} - h_1}{b_{11}^*}; g_2 = \frac{b_{11}^* \cdot v_{qs} + h_2}{b_{11}^*} \quad (52)$$

$$g_3 = -k_R \cdot x_{13} \cdot (x_2 - x_{12}) + \frac{k_R}{T_R} \cdot x_{14} \quad (53)$$

$$g_a = -k \frac{a_{31}^* \cdot a_{33}^*}{(a_{33}^*)^2 + (z_p \cdot g_3)^2}; g_b = k \frac{a_{31}^* \cdot z_p \cdot g_3}{(a_{33}^*)^2 + (z_p \cdot g_3)^2} \quad (54)$$

$$\hat{\omega}_{\lambda r} = z_p \cdot g_3 + a_{31}^* \cdot \frac{x_{12}}{x_{13}} \quad (55)$$

$$\alpha_{12}^a = \hat{\omega}_{\lambda r} + \alpha_{12} \cdot z_p \cdot x_5; \alpha_{34}^a = \hat{\omega}_{\lambda r} - \alpha_{34} \cdot z_p \cdot x_5 \quad (56)$$

Under these circumstances the mathematical model of the speed vector control system is fully determined as being defined by the non-linear differential equations system given by (34) whose initial condition is

$$x(0) = [p_1 \quad x_{130} \quad 0] \quad (57)$$

where:

$$p_1 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]; x_{130} = 10^{-9}$$

IV. THE ASYMPTOTIC STABILITY STUDY OF THE CONTROL SYSTEM.

In order to perform the analysis of the asymptotic stability of an induction motor based system, the following parameters have been considered:

- Electrical parameters: $P_N = 4 [KW]; U_N = 400 [V]$
 $R_s = 1.405 [\Omega]; R_r = 1.395 [\Omega]; L_s = 0.178039 [H];$
 $L_r = 0.178039 [H]; L_m = 0.1722 [H]; f_N = 50 [Hz].$
- Mechanical parameters: $n_N = 1430 [rot/min]; z_p = 2$
 $J = 0.0131 [kg \cdot m^2]; F = 0.002985 [N \cdot m \cdot s/rad].$

One of the main problems in the practical implementation of a speed control system for an induction motor is the controller tuning.

In present, the controllers tuning of the induction motors speed control systems is made only through experimental methods, and the time allocated for this type of tests is a really long one.

In order to design the automated controllers within the control system first we shall assume that the Extended Gopinath Observer is very well designed so that the estimated values are assumed to be equal to the real values of the motor.

Therefore, for the controllers composing block B2 of the speed control system the following analytical adjustment formulas are used [9], [10]:

- Current controller:

$$T_i = -\frac{1}{a_{11}^*}; K_i = \frac{1}{b_{11}^* \cdot T_{d1}^*} \quad (58)$$

- Flux controller:

$$T_\psi = T_r^*; K_\psi = \frac{T_r^*}{2 \cdot L_m^* \cdot T_{d1}^*} \quad (59)$$

- Torque controller:

$$T_M = T_{d1}^*; K_M = \frac{T_{d1}^*}{K_a \cdot |\psi_r^*| \cdot T_{d2}^*} \quad (60)$$

- Speed controller:

$$K_\omega = \frac{T_4 \cdot (1 + \rho^2)}{2 \cdot K_4 \cdot T_{d2}^*}; T_\omega = 4 \cdot \frac{T_{d2}^* \cdot (1 + \rho^2)}{(1 + \rho)^3} \quad (61)$$

where: $\rho = T_{d2}^* / T_4; K_4 = 1/F$ and $T_4 = J/F$.

The proportion and integration coefficients of the PI controller of the adapting mechanism of the Extended Gopinath Observer are determined using the linear equation of the estimation error [7].

- Adaptation mechanism:

$$k_R = \frac{1}{K_u \cdot T_{d1}^*}; T_R = \frac{T_r}{2}; K_u = a_{14}^* \cdot z_p \cdot |\psi_r^*|^2; k = 0.2 \quad (62)$$

In the above mentioned formulas, T_{d1}^* and T_{d2}^* are two time constancies imposed considering they need to respect the following conditions:

$$T_{d1}^* < T_r^*; T_{d2}^* < T_4 \text{ and } T_{d2}^* > T_{d1}^* \quad (63)$$

The tuning relations used are the ones presented in this paper and the constants that occur in the automated controllers tuning coefficients' calculus, are:

$$T_{d1}^* = 0.1 [ms]; T_{d2}^* = 0.75 [ms] \quad (64)$$

As the analysed control system, is non-linear, we can not speak of the system stability only about the equilibrium point's stability [11], [12], [13], [14], [15]. For this reason after solving the non-linear equation system:

$$f_i(x, u) = 0; i = \overline{1, 14} \quad (65)$$

obtained from the vectorial functions that define the system (34), we obtain the equilibrium point of the non-linear system. In order to solve the non-linear equation system (65) we shall apply Newton's method, and the equilibrium point obtained for an imposed input vector is noted like this:

$$b_{ma} = [b_{mai}]_{i=\overline{1, 14}}^T \quad (66)$$

Sequently we shall note by b_{ma} the equilibrium points' throng obtained for an input vector like:

$$u_{ma}^* = [u_{1m} \quad u_2 \quad u_{3a}]^T \quad (67)$$

The rotor angular speed within the input vector (67) is imposed based on the following mathematical relation:

$$u_{1m} = \omega_{rm}^* = n_m \frac{\pi}{60} \left[\frac{rad}{s} \right]; n_m = m \left[\frac{rot}{min} \right]; m = \overline{-n_N, n_N} \quad (68)$$

and the rotor flux modulus is given by the expression (69)

$$u_2 = \psi_r^* = \frac{T_s \cdot L_m}{L_s} \cdot \frac{U_N}{\sqrt{1 + T_r^2 \cdot z_p^2 \cdot \omega_{rn}^2}} [Wb] \quad (69)$$

The load torque within the input vector (67) is imposed

based on the following mathematical relation:

$$u_{3a} = a ; a = -\lfloor M_N \rfloor, \lfloor M_N \rfloor \quad (70)$$

where $\lfloor M_N \rfloor$ is the whole part of the expression (71).

$$M_N = P_N / \omega_{rN} [N \cdot m] ; \omega_{rN} = 2 \cdot \pi \cdot n_m / 60 [rad / s] \quad (71)$$

Sequently for the analysis of the stability we shall linearise the non-linear system (34), around the equilibrium point (66). The linearized model is given by the expression (72):

$$\dot{\Delta x}(t) = A_L \cdot \Delta x(t) + B_L \cdot \Delta u(t) \quad (72)$$

where A_L, B_L matrices are

$$A_L = \left[\frac{\partial f_i}{\partial x_j} (b_{ma}, u_{ma}^*) \right]_{i=\overline{1,14}; j=\overline{1,14}} \quad B_L = \left[\frac{\partial f_i}{\partial u_k} (b_{ma}, u_{ma}^*) \right]_{i=\overline{1,14}; k=\overline{1,3}}$$

Next, in order to study the asymptotic stability of the equilibrium points b_{ma} , the eigenvalues of the A_L matrix will be analyzed, so that if they have a strictly negative real part the b_{ma} equilibrium point is asymptotically stable for the linerized system (72). Under these circumstances according to the linearization theorem in a vicinity of the equilibrium point b_{ma} the non-linear system (34) is asymptotically stable. As the eigenvalues of the A_L matrix are presented within Figure 4, it results that the equilibrium points b_{ma} of the linerized system (72) are asymptotically stable and according to the linearization theorem the equilibrium points b_{ma} are asymptotically stable in certain vicinity for the non-linear system (34).

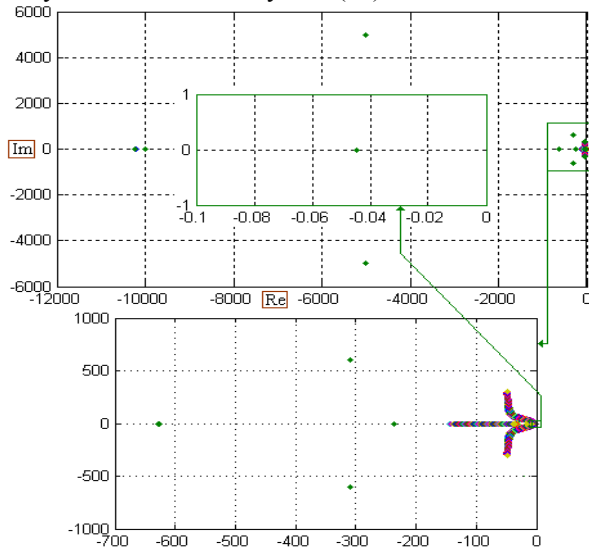


Figure 4. The eigenvalues of the matrix A_L

In order to determin the variation field of the identified rotor resistance for which the equilibrium points $b = b_{ma}$ remain asymptotic stable, we shall modify the identified rotor resistance of the motor based on the following mathematical relation:

$$R_r^* = \left(R_r + \frac{k}{100} \cdot R_r \right) [\Omega] ; k \in \mathbb{Z}_+ \quad (73)$$

where R_r is the resistance of the induction motor.

The tuning paramters of the automated controllers, within block B2, will be the same for all the testing period of the

structural stability, being determined based on the relations shown in this paper for a rotor resistance equal to the R_r resistance value of the induction motor. From those mentioned above we observe that for a specific input vector u_{ma}^* we shall have k equilibrium points. For this reason the equilibrium points will be noted: b_{mak} . On the other hand in order to determin the parametric stability field, for each equilibrium point b_{mak} we shall evaluate the eigenvalues of the matrix A_L , so that the field of these eigenvalues will be noted A_L^{mak} . The values of the coefficient k within the expression (73) for which the real part of the eigenvalues, that make up the A_L^{mak} field, becomes strictly positive, define the frontier of the parametric stability field of the identified rotor resistance of the induction motor.

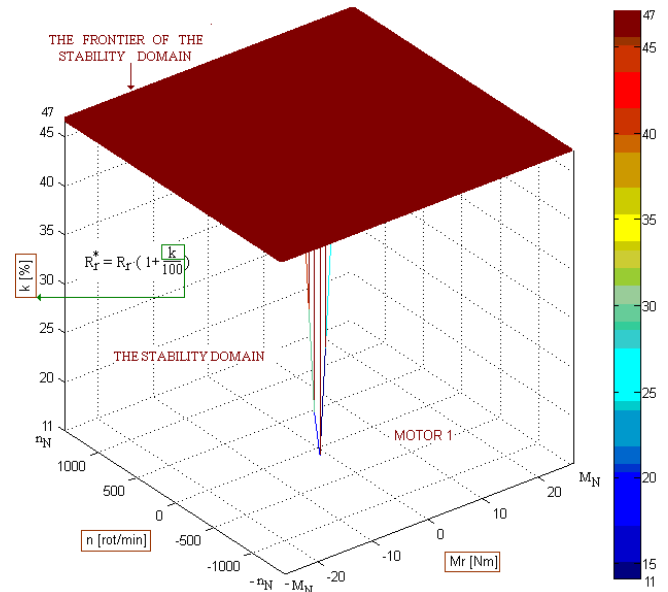


Figure 5. The stability parametric domain – continual case

The study of the stability control system for discrete case, suppose the discretization of the nonlinear system (34).

After the discretization, we get:

$$x(k+1) = f_a(x(k), u(k)) \quad (74)$$

where: $x(k) = [x_i(k)]^T$; $u(k) = [u_1(k) \ u_2(k) \ u_3(k)]^T$.

The equations defining the induction motor and equations defining the extended Gopinath observer will be discretization using the Euler method. The automatic controllers of the composition of the control system and the adaptation mechanism will be discretization using the Euler method. Thus we get:

$$f_{ai}(x, u) = x_i(k) + T_s \cdot f_i(x(k), u(k)) ; i = \overline{1,14} \quad (75)$$

where: T_s is sampling time.

As the analysed control system, is non-linear, we can not speak of the system stability only about the equilibrium point's stability. For this reason after solving the non-linear equation system:

$$f_{ai}(x, u) = x_i ; i = \overline{1,14} \Rightarrow f_i(x, u) = 0 ; i = \overline{1,14} \quad (76)$$

obtained from the vectorial functions that define the system (74), we obtain the equilibrium point of the non-linear system. Sequently for the analysis of the stability we shall

linearise the non-linear system (74), around the equilibrium point (66). The linearized model is given by the expression (77):

$$\Delta x(k+1) = F_D \cdot \Delta x(k) + H_D \cdot \Delta u(k) \quad (77)$$

where:

$$F_D = \left[\frac{\partial f_{ai}}{\partial x_j} (b_{ma}, u_{ma}^*) \right]_{j=1,14; j=1,14} \quad H_D = \left[\frac{\partial f_{ai}}{\partial u_k} (b_{ma}, u_{ma}^*) \right]_{j=1,14; k=1,3}$$

Proceeding in a similar manner the eigenvalues of the F_L matrix in case the entry vector is defined by (67) and the F_L matrix is obtained by using simplified digitization using a $T_s = 100 [\mu\text{sec}]$ sampling time are graphically presented in Figure 6.

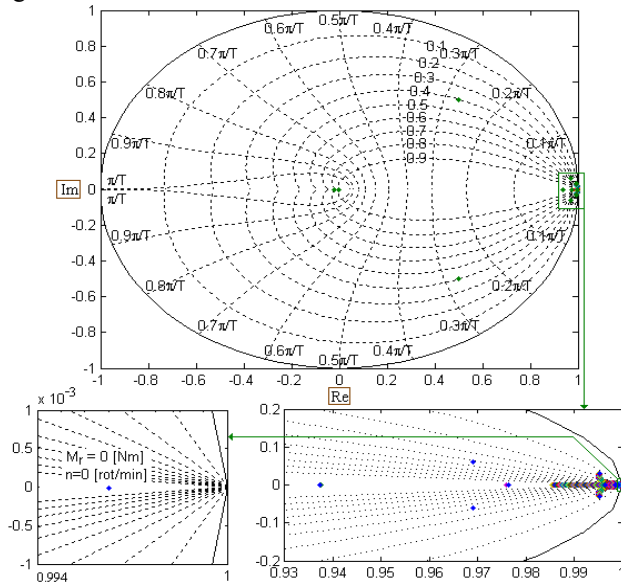


Figure 6. The eigenvalues of the matrix F_L

In order to determine the variation field of the identified rotor resistance for which the equilibrium points remain asymptotic stable, we shall modify the identified rotor resistance of the motor based on the relation mathematical (73). Preceding in a similar the parametric stability domain is:

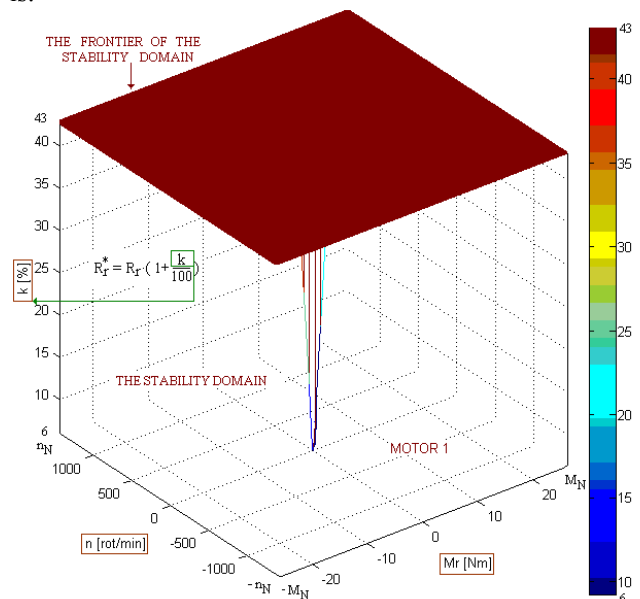


Figure 7. The stability parametric domain – discrete case

The results presented above have been partially proved by experiment, due to the limitations introduced by the inverter.

Next, the performances of the extended Gopinath estimator are presented in a variety of functional conditions.

Thus the image below will present the graphics for the real and estimated rotors fluxes and also the graphics for the imposed speed, real speed and the estimated speed for small, and medium imposed speeds.

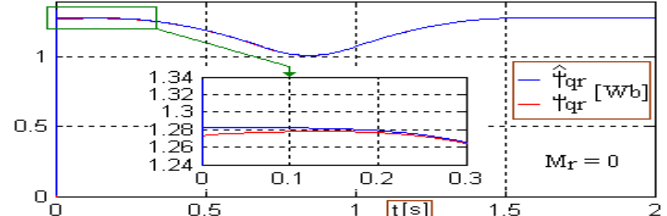


Figure 8. Real flux compared to the estimated flux $\omega_r^* = 5 \cdot \pi / 30 [\text{rad/s}]$

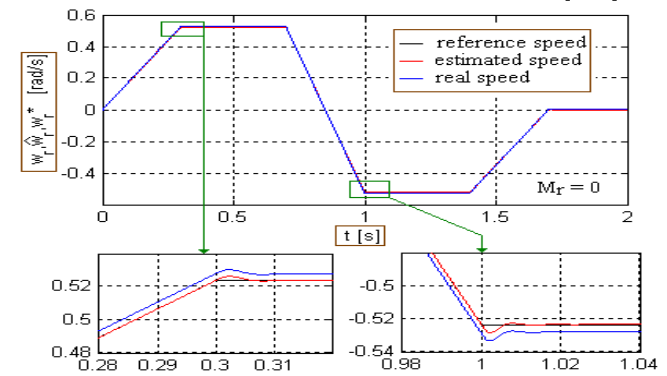


Figure 9. Real speed compared to the estimated speed and reference speed: $\omega_r^* = 5 \cdot \pi / 30 [\text{rad/s}]$

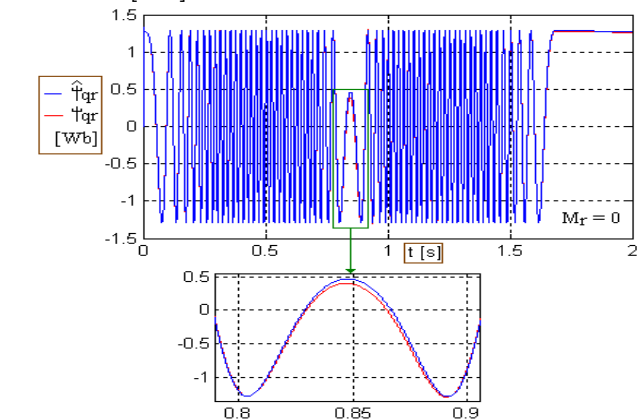


Figure 10. Real flux compared to the estimated flux:

$$\omega_r^* = 1430 \cdot \pi / 30 [\text{rad/s}]$$

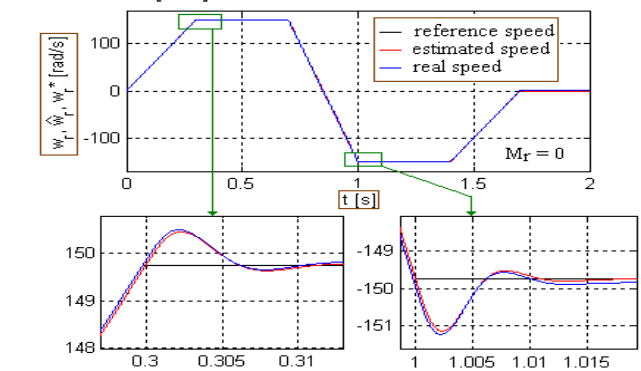


Figure 11. Real speed compared to the estimated speed and reference speed: $\omega_r^* = 1430 \cdot \pi / 30 [\text{rad/s}]$

On the other hand in the following image the effect of the rotors resistance will be presented, emphasizing the dynamic performances [16] of the EGO estimator. Thus the graphic

between the real and estimated rotor fluxes are shown as well as the graphic between the imposed speed, real speed and estimated speed for low and medium imposed speeds.

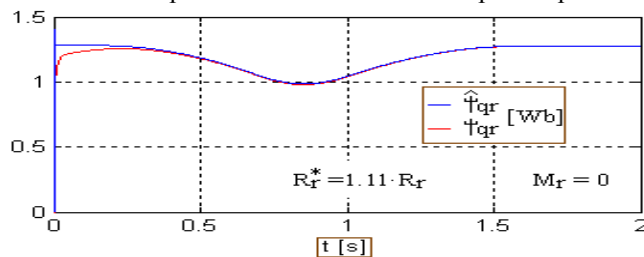


Figure 12. Real flux compared to the estimated flux: $\omega_r^* = 5 \cdot \pi / 30$ [rad/s]

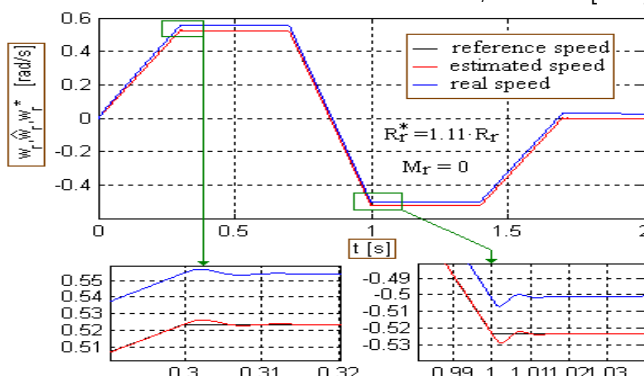


Figure 13. Real speed compared to the estimated speed and reference speed: $\omega_r^* = 5 \cdot \pi / 30$ [rad/s]

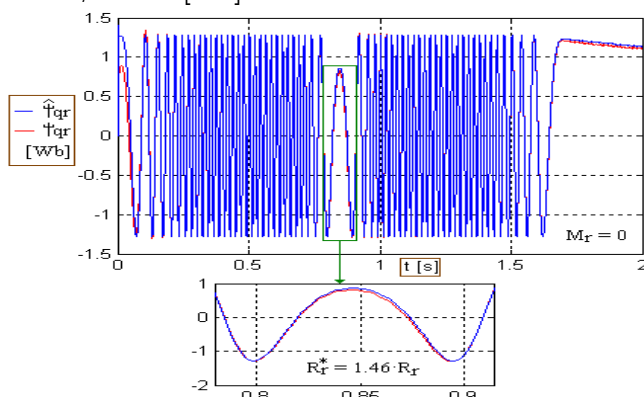


Figure 14. Real flux compared to the estimated flux:

$$\omega_r^* = 1430 \cdot \pi / 30 \text{ [rad/s]}$$

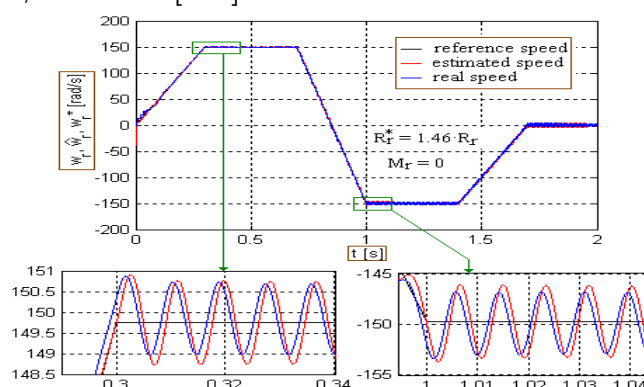


Figure 15. Real speed compared to the estimated speed and reference speed: $\omega_r^* = 1430 \cdot \pi / 30$ [rad/s]

V. CONCLUSION

This paper presents a new flux and rotor speed observer called an Extended Gopinath Observer (EGO). The design of the EGO observer is done based on an adaptive mechanism using the notion of Popov hyperstability.

The EGO observer, whose gate matrix is calculated with the relations (2), ensures the adjustment system very good dynamic performance that gives us the possibility to assert, that such an estimator could be successfully used in industrial applications.

The paper shows an analytic method of automated system controllers' tuning within an automated speed control system, for an induction motor.

The use of the controllers' tuning formula shown in this paper, has the advantage of ruling out the experimental methods used so far in the controllers' tuning within the speed vector control systems of an induction motor.

The controllers' designing, using the method presented in this paper, ensures the control system with a very good dynamics and robustness. These net advantages, recommend the successful use of this method in practice.

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