TCP Congestion Control for the Networks with Markovian Jump Parameters

Amir Hossein ABOLMASOUMI, Hamid Reza MOMENI
Automation Lab, Faculty of Electrical and Computer Engineering,
University of Tarbiat Modares, Tehran, Iran
momeni_h@modares.ac.ir

***Abstract*—This paper is concerned with the problem of TCP congestion control for the class of communication networks with random parameters. The linear dynamic model of TCP New Reno in congestion avoidance mode is considered which contains round trip delays in both state and input. The randomness of link capacity, round trip time delay and the number of TCP sessions is modeled with a continuous-time finite state Markov process. An Active Queue Management (AQM) technique is then used to adjust the queue level of the congested link to a predefined value. For this purpose, a dynamic output feedback controller with mode dependent parameters is synthesized to stochastically stabilize the TCP/AQM dynamics. The procedure of the control synthesis is implemented by solving a linear matrix inequality (LMI). The results are tested within a simulation example and the effectiveness of the proposed design method is verified.**

***Index Terms*—communication system traffic control, delay systems, linear matrix inequalities, Markov processes, random variables**

I. INTRODUCTION

With the fast growth and the wide usage of communication networks in recent years, large attention has been paid to the congestion control problem and active queue management (AQM) routers. Some of the most famous algorithms for the AQM goal are RED [1], PI [2] and adaptive PI [3]. For more AQM algorithms see [4-8]. Based on the measured queue length, q, at a congested router, the AQM algorithm computes a marking probability, p. Then the senders will be able to set their sending rates in order to achieve a desired queue level in the congested router. In [9], a dynamic model of TCP behavior in congestion avoidance mode is developed using fluid-flow and stochastic differential equation analysis. It is proved that this model accurately represents the dynamics of TCP. In [2], a simplified version of the model in [9] is used which ignores the TCP timeout mechanism.

The network conditions are mainly described by the parameters traffic load, round-trip time and link capacity. These three parameters could affect the traffic behavior. Increase or decrease in the traffic load (or number of active TCP sessions) could change the queue size in the bottleneck link. Round trip delay could also affect the stability and performance of the TCP/AQM dynamical model. The amount of link capacity limits the bandwidth of the total packet processing, so the variations of the link capacity can influence on the instantaneous queue length. The network conditions are usually varying in different daytimes, different applications and different services; therefore, tolerating different network conditions whereas preserving the desired performance is the key property of high quality AQM algorithms. See For example [3, 10, 11 and 12].

In addition to gently parameter changes, abrupt random changes in communication networks are not unusual cases. Such abrupt changes could move the equilibrium point in the linearized model, so the AQM methods which make use of the linearized model would show weak performances. The main idea of this paper is to stabilize the TCP/AQM systems with random network parameters. The parameters are modeled by a continuous-time finite state Markov process. Therefore, the TCP/AQM system can be represented as a delayed Markovian Jump System (MJS).

Markovian jump systems are some kinds of hybrid systems with their discrete state varying as a continuous-time finite state Markov process. These systems have been widely studied in the past years and many results on estimation and control problems, related to such systems, have been reported in the relevant literature (see for example [13-15]). The problem of Markovian jump systems with time delays is also considered in many researches in recent years. (See [16-23]).

To the knowledge of authors, the problem of TCP/AQM structure with Markovian changes in network parameters (such as link capacity, TCP load and round trip time) is not addressed in the literature. In this paper, a dynamic output controller is used to stabilize the Markovian jump TCP/AQM dynamics. By employing a Lyapunov-Krasovski functional, sufficient stochastic stability conditions are obtained in terms of LMIs. The controller gains are then synthesized from the LMIs. To prove the effectiveness of the approach, real TCP network conditions are simulated in two examples. It is shown that our approach could fix the queue length at a predefined value.

The rest of the paper is organized as follows: Section 2 describes the system model and some preliminaries about the Markovian jump system. Section 3 presents the main results on stability and dynamic output stabilization of the system, based on a LMI approach. The simulation example is given in Section 4 and finally Section 5 concludes the paper.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

In order to design controllers for the network congestion problem, TCP networks are simplified to a dumbbell topology shown in Figure 1. The network consists of N senders, a bottleneck router and a receiver. In this paper, like in [2], a simplified version of that model is used which

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neglects the TCP timeout mechanism. This model relates the average value of network variables and is described by the following coupled nonlinear differential equations:

$$\begin{cases} \dot{W}(t) = \dfrac{1}{R(r_t)} - \dfrac{W(t)}{2}\dfrac{W(t-R(r_t))}{R(t-R(r_t))}p(t-R(r_t)) \\ \dot{q}(t) = \begin{cases} -C + \dfrac{N(r_t)}{R(r_t)}W(t) & q>0 \\ \max\left(0, -C + \dfrac{N(r_t)}{R(r_t)}W(t)\right), & q=0 \end{cases} \end{cases} \tag{1}$$

The linearized version of the TCP new Reno model is described by the following coupled delay differential equations with Markovian jump parameters:

$$\begin{cases} \delta\dot{W}(t) = \dfrac{-N(r_t)}{R^2(r_t)C(r_t)}\left(\delta W(t) + \delta W(t-R(r_t))\right) \\ -\dfrac{1}{R^2(r_t)C(r_t)}\left(\delta q(t) + \delta q(t-R(r_t))\right) \\ -\dfrac{R(r_t)C^2(r_t)}{2N^2(r_t)}\delta p(t-R(r_t)) \\ \delta\dot{q}(t) = \dfrac{N(r_t)}{R(r_t)}\delta W(t) - \dfrac{1}{R(r_t)}\delta q(t) \end{cases} \tag{2}$$

in which *W* represents average TCP window size. *q* stands for the average queue length, *R* is round-trip time, C is the available capacity of congested link, N represents the load factor (number of TCP sessions) and *p* is the probability of packet marking. Also we have $\delta W(t) = W - W_0(r_t)$, $\delta q(t) = q - q_0(r_t)$ and $\delta p(t) = p - p_0(r_t)$ where $(W_0(r_t), q_0(r_t), p_0(r_t))$ is the equilibrium point of the system and:

$$\begin{aligned} & W_0^2(r_t)p_0(r_t) = 2, \\ & W_0(r_t) = \frac{R_0(r_t)C(r_t)}{N(r_t)}, \\ & R_0(r_t) = \frac{q_0(r_t)}{C(r_t)} + T_p(r_t) \end{aligned} \tag{3}$$

$T_p(r_t)$ is the propagation delay. Note that we ignore the dependence of the time-delay argument on queue-length. It is mostly because the time for a packet to pass a bottleneck is negligible when compared with the propagation delay of the whole path of the packet. In other words, we take $R_0(r_t) \approx T_p(r_t)$.

In (2), (3) , $\{r_t\}$ is a continuous-time homogenous Markovian process with right continuous trajectories taking values in a finite set $S = \{1, \ldots, N\}$ with transition probability matrix $\Lambda = [\lambda_{ij}]$ given by:

$$\begin{cases} P[r_{t+h} = i \mid r_t = j] = \begin{cases} \lambda_{ij}h + o(h) & i \neq j \\ 1 + \lambda_{ii}h + o(h) & otherwise \end{cases} \\ \lim\limits_{h\to 0}\dfrac{o(h)}{h} = 0 \quad , \quad \lambda_{il} \geq 0 \quad , \lambda_{ii} = -\sum_{l\neq i}\lambda_{il} \end{cases} \tag{4}$$

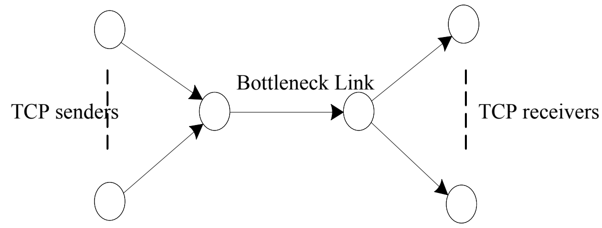

Figure 1: Network Topology

For the simplicity in notation, when necessary, we refer to the Markov process r_t by i.

The round trip delay values $\{R_1, ..., R_N\}$ are considered to be limited in the interval $[\underline{R}, \overline{R}]$.

System (2) describes a linear model of a congested link under TCP Reno source side algorithm, in which, the equilibrium is considered to be a stochastic point. However the nonlinear model of (1) itself is obtained using fluid-flow and stochastic differential equation analysis [9]. In fact, (2) represent the behavior of the TCP/AQM system when additional statistic data as the Markov probability rates are provided. These rates can be calculated experimentally in the communication networks.

System (2) could be represented as follows:

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{di}x(t-R_i) + B_i u(t-R_i) \\ y(t) = Cx(t) \\ 0 < u < 1 \end{cases} \tag{5}$$

in which $x(t) = \left[\delta W^T(t) \quad \delta q^T(t)\right]^T$ and

$$A_i = \begin{pmatrix} \dfrac{-N_i}{R_i^2C_i} & \dfrac{-1}{R_i^2C_i} \\ \dfrac{-N_i}{R_i} & \dfrac{-1}{R_i} \end{pmatrix}, \quad A_{di} = \begin{pmatrix} \dfrac{-N_i}{R_i^2C_i} & \dfrac{-1}{R_i^2C_i} \\ 0 & 0 \end{pmatrix}, \quad B_i = \begin{pmatrix} \dfrac{-R_iC_i^2}{2N_i^2} \\ 0 \end{pmatrix} \tag{6}$$

System (5) is a Markovian jump delay system with mode dependent state and input delays. Also the input is restricted to be between zero and one since it is a probability function for marking/dropping of the packets.

We take $y(t) = \delta q(t)$ as the output of the system. In AQM, the probability of packet marking is calculated based on this queue length.

Definition 1: The Markovian jump system

$$\dot{x}(t) = A_i x(t) + A_{di} x(t-\tau_i)$$
$$x(t) = \phi(t) \quad \forall t \in [-\bar{\tau}, 0] \tag{7}$$

is stochastically stable for any initial state in a neighborhood of origin, if the following holds for any initial conditions φ, r_0 :

$$E\left(\int_0^\infty \|x(t)\|^2 dt \mid \varphi, r_0\right) \le \infty \tag{8}$$

III. MAIN RESULTS

In this section, a delay-dependent stochastic stability condition for the closed loop system will be developed and the controller gains will be obtained. First we introduce the AQM dynamic output feedback controller which should stabilize the TCP/AQM structure. We assume the controller to be:

$$\dot{x}_c(t) = K_{Ai} x_c(t) + K_{Bi} \delta q(t)$$
$$\delta p(t) = K_{Ci} x_c(t) \tag{9}$$

For the simplicity in solving LMIs we choose K_{Ci} to be known constant matrix. The matrices K_{Ai} and K_{Bi} will be computed from the LMI conditions.

Assuming the above dynamic output controller and some basic calculations, the closed loop system can be obtained as follows:

$$\dot{\eta}(t) = \tilde{A}_i \eta(t) + \tilde{A}_{di} \eta(t-R_i) \tag{10}$$

where

$$\eta(t) = \begin{bmatrix} x^T(t) & x_c^T(t) \end{bmatrix}^T$$
$$= \begin{bmatrix} \delta W^T(t) & \delta q^T(t) & x_c^T(t) \end{bmatrix}^T \tag{11}$$

and

$$\tilde{A}_i = \begin{pmatrix} A_i & 0 \\ K_{Bi} & K_{Ai} \end{pmatrix},$$
$$\tilde{A}_{di} = \begin{pmatrix} A_{di} & B_i K_{Ci} \\ 0 & 0 \end{pmatrix} \tag{12}$$

The following lemma represents a stochastic stability criterion for the Markovian jump systems with mode-dependent delays. It is a generalized version of lemma 3 in [24] for the delay Markovian jump systems with mode dependent delays.

Lemma 1*:* System (10) is stochastically stable if there exist symmetric and positive-definite matrices P_i, Z_i, Q, Z and the matrices T_{1i}, T_{2i} and T_{3i} ($i \in S$) such that

$$\sum_{j\in S} \lambda_{ij} Z_j < Z$$
$$W_i + T_i^T \bar{A}_i + \bar{A}_i^T T_i + \Gamma^T Y_i + Y_i^T \Gamma + R_i X_i < 0 \tag{13}$$
$$\begin{bmatrix} X_i & Y_i \\ Y_i^T & Z_i \end{bmatrix} > 0$$

where

$$W_i = \begin{bmatrix} \rho_i Q + \sum_{j\in S} \lambda_{ij} P_j & 0 & P_i \\ 0 & -Q & 0 \\ P_i & 0 & \bar{R} Z_i + \frac{1}{2}\bar{R}^2 Z \end{bmatrix}, \tag{14}$$
$$T_i = \begin{bmatrix} T_{1i} & T_{2i} & T_{3i} \end{bmatrix}$$

and

$$\rho_i = 1 + |\lambda_{ii}|(\bar{R} - \underline{R}) \tag{15}$$

Proof: Take the stochastic Lyapunov functional $V(\cdot): R^{2+n_c} \times R_+ \times S_1 \to R_+$ to be:

$$V(\eta(t), i) = V_1(\eta(t), i) + V_2(\eta(t), i)$$
$$+ V_3(\eta(t), i_t) + V_4(\eta(t), i) \tag{16}$$
$$+ V_5(\eta(t), i)$$

in which

$$V_1(\eta(t), r_t) = \eta^T(t) P_i \eta(t)$$
$$V_2(\eta(t), r_t) = \int_{t-R_i}^t \eta^T(s) Q \eta(s) ds$$
$$V_3(\eta(t), r_t) = \int_{-\bar{R}}^0 \int_{t+\theta}^t \dot{\eta}^T(s) Z_i \dot{\eta}(s) ds d\theta \tag{17}$$
$$V_4(\eta(t), r_t) = |\lambda_{ii}| \int_{-\bar{R}}^{-\underline{R}} \int_{t+\theta}^t \eta^T(s) Q \eta(s) ds d\theta$$
$$V_5(\eta(t), r_t) = \int_{-\bar{R}}^0 \int_{t+\theta}^t \dot{\eta}^T(s) Z_i \dot{\eta}(s)(s - t - \theta) ds d\theta$$

Defining a new process $\{(\eta(t), r_t)\}$ by $\eta_t(\theta) = \eta(t+\theta)$ for each $\theta \in (-\bar{R}, 0)$ the weak infinitesimal operator $\mathcal{A}$ of the stochastic process $\{(x(t), r_t)\}$ is given by

$$\mathcal{A}V(\eta(t), r_t) =$$
$$\lim_{h\to 0} \frac{1}{h}\left[E\{V(\eta(t+h), r_{t+h}) \mid \eta(t), r_t\} - V(\eta(t), r_t)\right] \tag{18}$$

Then, for each $r_t = i$ ($i \in S$) we have

$$\mathcal{A}V_1(\eta(t), i, j) =$$
$$2\eta^T(t) P_i \dot{\eta}(t) + \eta^T(t)\left[\sum_{j\in S} \lambda_{ij} P_j\right]\eta(t)$$
$$+2\left(T_{1i}\eta(t) + T_{2i}\eta(t-R_i) + T_{3i}\dot{\eta}(t)\right)\left(\tilde{A}_i \eta(t) - \dot{\eta}(t)\right. \tag{19}$$
$$\left. - \tilde{A}_{di} \int_{t-R_i}^t \dot{\eta}(s) ds\right)$$

for any weighting matrices T_{1i}, T_{2i} and $T_{3i} \in R^{(2+n_c)\times(2+n_c)}$. If we define

$$\zeta(t) = \begin{bmatrix} \eta^T(t) & \eta^T(t-R_i) & \dot{\eta}^T(t) \end{bmatrix}^T,$$
$$\bar{A}_i = \begin{bmatrix} \tilde{A}_i & \tilde{A}_{di} & -\mathrm{I} \end{bmatrix}, \tag{20}$$
$$\Gamma = \begin{bmatrix} \mathrm{I} & -\mathrm{I} & 0 \end{bmatrix}$$

then we have

$$\mathcal{A}V_1\left(\eta(t),r_t\right)\leq\zeta^T(t)\begin{bmatrix}\sum_{j\in S}\lambda_{ij}P_j & 0 & P_i\\ 0 & 0 & 0\\ P_i & 0 & 0\end{bmatrix}\zeta(t) \tag{21}$$

$$+2\zeta^T(t)T_i^T\bar{A}_i\zeta(t)+R_i\zeta^T(t)X_i\zeta(t)$$

$$+2\zeta^T(t)Y_i\Gamma\zeta(t)+\int_{-R_i}^{0}\dot{\eta}^T(t+s)Z_i\dot{\eta}(t+s)ds$$

Moreover,

$$\begin{aligned}\mathcal{A}V_2\left(\eta(t),i\right)&\leq\eta^T(t)Q\eta(t)-\eta^T(t-R_i)Q\eta(t-R_i)\\&+|\lambda_{ii}|\int_{t-\bar{R}}^{t-\underline{R}}\eta^T(s)Q\eta(s)ds\\ \mathcal{A}V_3\left(\eta(t),i\right)&=\bar{R}\dot{\eta}^T(t)Z_i\dot{\eta}(t)\\&-\int_{-\bar{R}}^{0}\dot{\eta}^T(t+s)Z_i\dot{\eta}(t+s)ds\\&+\int_{-\bar{R}}^{0}\int_{t+\theta}^{t}\dot{\eta}^T(s)\left(\sum_j\lambda_{ij}Z_j\right)\dot{\eta}(s)\\ \mathcal{A}V_4\left(\eta(t),i\right)&=|\lambda_{ii}|\left(\bar{R}-\underline{R}\right)\eta^T(t)Q\eta(t)\\&-|\lambda_{ii}|\int_{t-\bar{R}}^{t-\underline{R}}\eta^T(s)Q\eta(s)ds\\ \mathcal{A}V_5\left(\eta(t),i\right)&=\frac{1}{2}\bar{R}^2\dot{\eta}^T(t)Z\dot{\eta}(t)-\int_{-\bar{R}}^{0}\int_{t+\theta}^{t}\dot{\eta}^T(s)Z\dot{\eta}(s)\end{aligned} \tag{22}$$

which yields $\mathcal{A}V\left(\zeta(t),i\right)\leq 0$ if (13) holds. Thus, there exist positive scalars δ_i, $i\in S$ for which

$$\mathcal{A}V\left(\zeta(t),i\right)\leq-\delta_i\left\|\zeta(t)\right\|^2 \tag{23}$$

The rest of the proof is straightforward using Dynkin's formula and the stochastic stability theory. (See [24]).

■■■

Now we are in the position to solve the stabilization problem of the TCP/AQM system with Markovian jump parameters. Based on Lemma 1, we can obtain a dynamic output feedback controller in the form of (9), However, (13) is not linear in terms of variables. Some techniques should be applied to (13) to convert it to LMI conditions. The following theorem gives the results on the controller synthesis.

Theorem1: Consider the TCP/AQM dynamics as a Markovian jump system in (10). The closed loop is stochastically stable if there exist symmetric and positive definite matrices P_i, Z_i, Q, Z and the matrices H_{1i} and H_{2i} ($i\in S$) such that

$$\begin{aligned}&\sum_{j\in S}\lambda_{ij}Z_j<Z\\&\bar{W}_i+\Gamma^TY_i+Y_i^T\Gamma+R_iX_i<0\\&\begin{bmatrix}X_i & Y_i\\ Y_i^T & Z_i\end{bmatrix}>0\end{aligned} \tag{24}$$

where

$$\bar{W}_i=W_i+\begin{bmatrix}G_i+G_i^T & G_i+G_{di}^T & G_i-T_{1i}^T\\ * & G_{di}+G_{di}^T & G_{di}^T-T_{1i}^T\\ * & * & -T_{1i}-T_{1i}^T\end{bmatrix}$$

$$T_{1i}=\begin{bmatrix}T_{11i} & 0\\ 0 & T_{12i}\end{bmatrix}$$

and

$$G_i=\begin{bmatrix}A_i^TT_{11i} & C^TH_{1i}\\ 0 & H_{2i}\end{bmatrix},$$

$$G_{di}=\begin{bmatrix}A_{di}^TT_{11i} & 0\\ K_{Ci}^TB_i^TT_{11i} & 0\end{bmatrix}$$

Also the controller gains that stabilize system (10) in the stochastic sense are given by

$$\begin{aligned}K_{Ai}^T&=H_{2i}T_{12i}^{-1},\\ K_{Bi}^T&=H_{1i}T_{12i}^T\end{aligned} \tag{25}$$

Note that, as we said later, the matrix K_{Ci} is assumed to be a known constant matrix.

Proof: The term $T_i^T\bar{A}_i+\bar{A}_i^TT_i$ in (13) can be written as follows:

$$T_i^T\bar{A}_i+\bar{A}_i^TT_i=$$

$$\begin{bmatrix}T_{1i}^T\tilde{A}_i+\tilde{A}_i^TT_{1i} & T_{1i}^T\tilde{A}_{di}+\tilde{A}_i^TT_{2i} & \tilde{A}_i^TT_{3i}-T_{1i}^T\\ * & T_{2i}^T\tilde{A}_{di}+\tilde{A}_{di}^TT_{2i} & \tilde{A}_{di}^TT_{3i}-T_{2i}^T\\ * & * & -T_{3i}^T-T_{3i}\end{bmatrix} \tag{26}$$

We take $T_{1i}=T_{2i}=T_{3i}$. The terms $G_i=\tilde{A}_i^TT_{1i}$ and $G_{di}=\tilde{A}_{di}^TT_{1i}$ become:

$$\begin{aligned}\tilde{A}_i^TT_{1i}&=\begin{bmatrix}A_i^TT_{11i} & C^TK_{Bi}^TT_{12i}\\ 0 & K_{Ai}^TT_{12i}\end{bmatrix},\\ \tilde{A}_{di}^TT_{1i}&=\begin{bmatrix}A_{di}^TT_{11i} & 0\\ K_{Ci}^TB_i^TT_{11i} & 0\end{bmatrix}\end{aligned} \tag{27}$$

and the proof is complete.

■■■

Remark: The term $K_{Ci}^TB_i^TT_{11i}$ causes problem in computing the gain K_{Ci} from LMIs, since the variables T_{11i} and K_{Ci} are multiplied. In order to obtain a LMI condition, one can use an equality constraint like $B_i^TT_{11i}=\hat{T}_{11i}B_i^T$. Then, the multiple term $K_{Ci}^T\hat{T}_{11i}$ can be computed from the LMIs. In TCP dynamics, because of a zero element in B_i, the equality does not have any solution. Although, a suitable transformation can be found to transform the system into a system with complete B_i. Here, for the simplicity, we choose K_{Ci} to be a known constant matrix. The matrices K_{Ai} and K_{Bi} will be computed from the LMI conditions.

IV. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed hybrid dynamic output feedback algorithm (HDOF) by a number of simulations performed using MATLAB. The performance of HDOF is compared with PI [2] which is a well known AQM algorithm. A single bottlenecked router is considered running our AQM method. We choose the number of TCP sessions to be $N_1 = 50, N_2 = 70, N_3 = 90$, also we select the other network parameters as follows:

$$\begin{aligned} &C_1 = 3500, C_2 = 3700, C_3 = 3900 \ \ packet/s \\ &T_{p1} = 100ms, T_{p2} = 200ms, T_{p3} = 300ms \end{aligned} \tag{28}$$

First, the transition rates of the Markov process is chosen such that it reflects the slow network variations:

$$\Lambda_{slow} = \begin{bmatrix} -3\times10^{-6} & 2\times10^{-6} & 10^{-6} \\ 2\times10^{-6} & -4\times10^{-6} & 2\times10^{-6} \\ 6\times10^{-6} & 4\times10^{-6} & -10^{-5} \end{bmatrix} \tag{29}$$

The desired queue length is selected to be 200 packets. By solving the LMIs in (24) the following controller gains is obtained:

$$\begin{aligned} &K_{A1} = \begin{bmatrix} -2.1162 & 0 \\ 0 & -2.1162 \end{bmatrix}, \\ &K_{A2} = \begin{bmatrix} -2.1114 & 0 \\ 0 & -2.1114 \end{bmatrix}, \\ &K_{A3} = \begin{bmatrix} -2.1298 & 0 \\ 0 & -2.1298 \end{bmatrix}, \\ &K_{B1} \simeq \begin{bmatrix} 1.80\times10^{-9} \\ 0 \end{bmatrix}, \\ &K_{B2} \simeq \begin{bmatrix} 2.53\times10^{-9} \\ 0 \end{bmatrix}, \\ &K_{B3} \simeq \begin{bmatrix} 5.66\times10^{-9} \\ 0 \end{bmatrix}, \\ &K_{C1} = K_{C2} = K_{C3} = \begin{bmatrix} 0.001 & 0 \end{bmatrix}. \end{aligned} \tag{30}$$

The TCP dynamics are assumed to be in the mode 1 at the beginning. The gains for PI controller are chosen to be $K_p = 1.822\times10^{-5}, K_i = 1.816\times10^{-5}$ for the network parameters defined in mode 1. Figure 2 shows the mode variations and the queue length when using both AQM methods HDOF and PI. It is observed that the AQM method can effectively regulate the queue length in the desired value.

The performance of our approach in networks with average and fast jumps is shown in figures 3, 4. The chosen transition rates are as follows:

$$\begin{aligned} &\Lambda_{average} = \begin{bmatrix} -2\times10^{-5} & 10^{-5} & 10^{-5} \\ 3\times10^{-5} & -7\times10^{-5} & 4\times10^{-5} \\ 7\times10^{-6} & 3\times10^{-6} & -10^{-5} \end{bmatrix}, \\ &\Lambda_{fast} = \begin{bmatrix} -10^{-4} & 5\times10^{-5} & 5\times10^{-5} \\ 10^{-4} & -3\times10^{-4} & 2\times10^{-4} \\ 7\times10^{-5} & 3\times10^{-5} & -10^{-4} \end{bmatrix}. \end{aligned} \tag{31}$$

Note that all of three assumed conditions could be realistic depending on the daytimes, the behavior of the network users and the current capacity and delay of the network.

As seen in figures 3, 4, the dynamic output feedback controller, as an AQM method, could almost overcome the average and even hard network conditions; however, the queue adjustment is not very precise. It is mostly because the settling time is more than one mode duration time. In both methods, the queue stabilizes rapidly, but as we expected, the PI controller cannot overcome the abrupt changes in network parameters. The performance of PI is even worse when the changes are more rapidly. Totally, it can be seen from Figure 2-4 that the queue is better adjusted using HDOF method.

V. Conclusion

In this paper, we have investigated the problem of designing a hybrid output feedback controller for a TCP congestion control problem modeled by a delay Markovian jump system. Based on Lyaponuv-Krasovski functionals, LMI-based sufficient conditions for the stochastic stability of the TCP/AQM were derived. A desired controller then was constructed by solving these LMIs. We simulated the networks dynamics using our AQM method and the well known PI method in different parameter change conditions. It was shown that different network conditions, the hybrid output feedback controller can effectively stabilize the queue length. Future work will consider the extension of the proposed approach from the model of a single bottleneck link to the case of multiple bottleneck links.

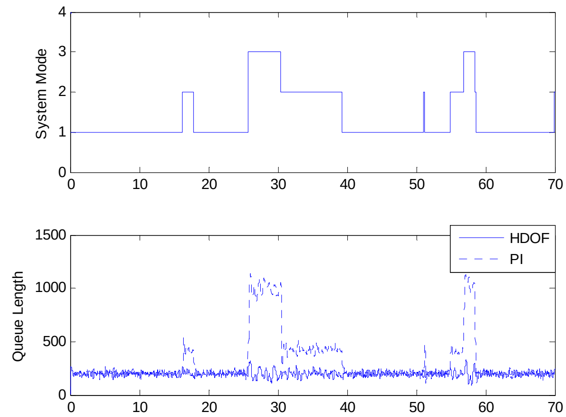

Figure 2: Mode variations and queue length for networks with infrequent parameter jumps

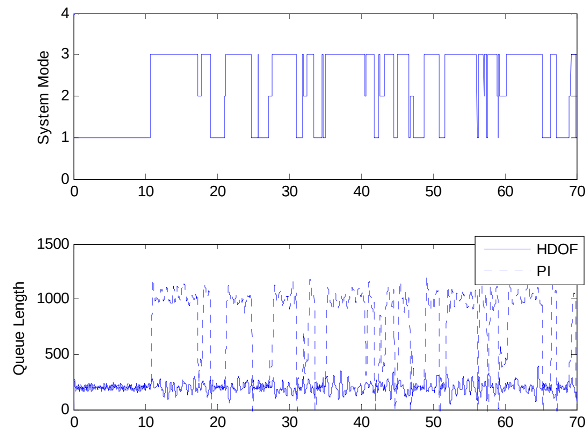

Figure 3: Mode variations and queue length for networks with average parameter jumps

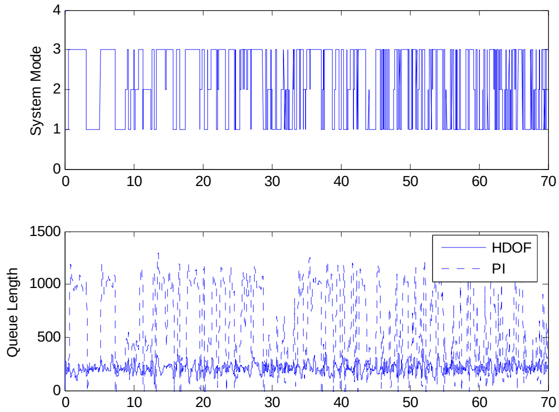

Figure 4: Mode variations and queue length for networks with heavy parameter jumps

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