# Genetic Algorithm based Servo System Parameter Estimation during Transients

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Abstract—The application of Genetic Optimization Algorithm in estimation of the parameters of servo electrical drives is proposed. In comparison with this planned method, least squared error (LSE) estimation method is considered as an expedient method for parameter estimation. Regardless of LSE estimation, Genetic Algorithm method is not restricted to the linear systems with respect to the parameters. GA is imported as an optimization method in comparison with conventional optimization methods because of its power in searching whole solution space with more probability to finding the global optimum. As a condition for convergence, transient excitation is considered instead of persistent excitation. Finally, comparison between LSE and GA based parameter estimation is presented to indicate robustness and resolution of GA identification method. It will be shown that the GA method of estimation has better results in the startup and transients of the system where there is a lack of persistent excitation.

Index Terms—Parameter Estimation, transient response, Genetic optimization, System Identification, Servo drive

# I. INTRODUCTION

Servo drives are widely used as positioning systems in low power industrial applications. In practice, where real data are used, the driver parameters and the system parameters are unknown or varying with load conditions. In this paper, the dynamic treatment of Servo Motor with its driver is considered as the system that must be identified with proposed parameter estimation method [1].

The system identification can be carried out as non-parametric or parametric models. Non-parametric models correspond to such models which are described by a function, curve or table. However, in many cases, it is relevant to deal with parametric models. Parametric vectors prescribe such models, which will be denoted by  $\theta$ . We obtain the structure of dynamic model of servo motor when  $\theta$  is varied over some set of feasible values [1].

In general, the experimental condition is a description of how the identification experiment is carried out. This includes the selection and generation of the sampling interval, the input signal, pre filtering of data prior to estimation of the parameters, etc. The experimental condition is determined when the characteristic property of system cannot be changed by the user during data collection.

Recursive LSE and Genetic Algorithm Estimation methods are considered as parametric methods. The performance of LSE method and the genetic algorithm optimization in identifying the dynamic state of servo are compared together. As we see, the proposed GA method shows better estimation of the system parameters during startup while the LSE cannot converge during such a short

period. For better explanation of GA algorithm applied to drive systems, the reader is referred to references [6]-[8].

In this paper, we will introduce the application of genetic algorithm optimization in parametric model identification. Minimizing the error function is the key element in obtaining the unknown parameters. The GA must formulate a fitness function according to the sum-squared error in order to treat the parameter estimation problem. The proposed algorithm begins with a collection of parameter estimates (chromosomes), each one being evaluated for its fitness in solving the given optimization task. In each generation, the chromosomes with higher fitness values are allowed to mate and bear offspring. The children that are new parameter estimates form the basis for the next generation. The use of crossover and mutation causes this algorithm to find the global optimum solution without being trapped in local minimum. GA has been successfully applied to a variety of optimization problems, such as image processing and fuzzy logic controller design [1-2-3]. As it is explained, the transient excitation instead of the persistent excitation in the startup period of the servo system leads to better parameter estimation with GA based method of estimation.

# II. DESCRIPTION OF THE METHOD

In general systems, assuming that y(t) can be measured in general form, the model can be calculated as the product of the variables and motor parameters. This model may be nonlinear with respect to the parameters. After calculating  $\hat{y}(t)$  corresponding to unknown parameters, the following model is applicable.

In each sample time  $T_s$ ,  $y(kT_s)$  is real number and  $\hat{y}(kT_s)$  is a function of unknown parameters in that sample time. The estimated model can be obtained with any information about the location of poles or guessing the dynamic state of model.

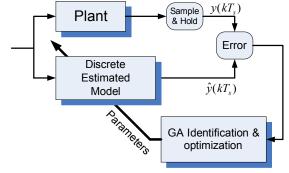


Figure 1. Estimation process.

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The aim is to minimize the error function square and to obtain unknown parameters. Consider J in the following formula:

$$J = \sum_{k=0}^{N} (y(kT_s) - \hat{y}(kT_s))^2$$
 (1)

as a fitness function to be minimized. Using Genetic Algorithm Optimization methods for minimizing fitness function, the parameters are estimated during transients or startup. In many situations, we can specify some restrictions for the model parameters, which makes the problem more complicated.

In this method, according to transient excitation, the settling time is definite and equals  $t_s = NT_s$ , where N is the number of samples, which will be obtained if the sampling frequency is definite  $(f_s = T_s^{-1})$ .

## III. PROBLEM STATEMENT

We intend to simulate some servo by estimating the dynamic state of Motor. The state equations of the Motor system are written as follows [4]:

$$\frac{di_{a}(t)}{dt} = -\frac{R_{a}}{L_{a}}i_{a}(t) - \frac{K_{b}}{L_{a}}\omega_{m}(t) + \frac{1}{L_{a}}e_{a}(t)$$
(2)

$$\frac{d\omega_m(t)}{dt} = \frac{K_T}{J_m} i_a(t) - \frac{B_m}{J_m} \omega_m(t) - \frac{1}{J_m} T_L(t)$$
 (3)

Where  $i_a(t)$  is armature current,  $\omega_m(t)$  is rotor angular velocity and the parameters  $R_a, L_a, K_b, K_T, B_m, J_m$  are Motor parameters that are described in Table I.

According to [4], dynamic treatment of Motor can be modeled as follows:

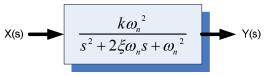


Figure 2. Second order dynamic model.

 $\xi$  is referred to as the damping ratio, the parameter  $\omega_n$  is the un-damped natural frequency and k, is the gain of system. We want to identify these three parameters by minimizing the sum of squared errors, as in Figure 2.

Now, consider a tested motor with the parameters as shown in Table I.

TABLE I. TESTED MOTOR PARAMETERS

Item	Value
Resistance $R_a$	$0.8\Omega$
Inductance L	$5 \times 10^{-3} H$
Back-emf constant K <sub>b</sub>	$8\times10^{-3} V/rpm$
Torque constant K <sub>T</sub>	$8\times10^{-3} N.m/Amp$
Rotor inertia J	1.5×10 <sup>-5</sup> N.M.S
Friction coefficient f	$2.5 \times 10^{-5} N.M / rpm$

The model transfer function is calculated as in Fig. 2.

$$H(s) = \frac{a_{2k}s + a'_{2k}}{s^2 + b_{2k}s + b'_{2k}} = \frac{k\omega_n^2}{s^2 + 2\eta\omega_n s + \omega_n^2}$$
(4)

Where H(s) is Estimated Model transfer function. Then, we have the system impulse response as:

$$\hat{h}(t) = \left\{ \frac{k\omega_n^2}{s^2 + 2\eta\omega_n s + \omega_n^2} \right\}^{-1}$$

$$= \frac{k\omega_n}{\sqrt{1 - \eta^2}} e^{-\eta\omega_n t} \sin(\omega_n \sqrt{1 - \eta^2} t)$$
(5)

$$\hat{y}(t) = \hat{h}(t) * u(t) = k(1 - \frac{1}{\sqrt{1 - \eta^2}} e^{-\eta \omega_n t} \sin(\omega_n \sqrt{1 - \eta^2} t + \cos^{-1} \eta)$$
(6)

Consider the response of experienced Servo Motor to step function:

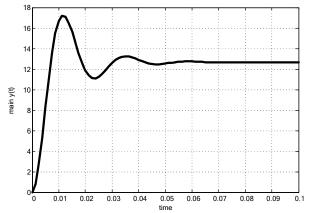


Figure 3. Step response of tested Motor.

Currently, Genetic Algorithm methods are used for finding unknown parameters. In the last part of this paper, Least Squared Estimation (LSE) is used in comparison with the proposed method. The gradient base LSE is used according to transient excitation. To simplify the comparison, we consider only two unknown parameters,  $\eta$  and k, and assume that  $\omega_n$  is determined. ( $\omega_n = 283.5$ )

# IV. PARAMETER ESTIMATION VIA GENETIC ALGORITHM

The genetic algorithm is a stochastic optimization algorithm that was originally motivated by the mechanisms of natural selection and evolution of genetics. In the following, a parameter estimation algorithm is developed based on GA evaluation of the unknown parameters, by carrying out minimization of the sum squared errors in (1). GA

Operators are listed in the following paragraphs[6]-[8].

Genetic algorithm is effective when used with its best operations and parameter values. The following operators are modified due to the experimental results.

- 1- The fitness function (J) is considered as (1) to be optimized as the objective function.
- 2- The population size determines the size of the population at each generation. Choosing the population size of 50 will satisfy the results. The population can be represented by 50 by 2 matrix, while this optimization includes two variables. In each iteration, a series of computations on the current population is performed by the genetic algorithm to produce a new population.

The first step in the algorithm is to create a random initial population in the interval of [0, 2.5], as initial range.

- 3- At each step, the genetic algorithm is used by the current population to create the children that make up the next generation. Individuals with better fitness value are usually selected by algorithm. Selection mechanism has uniform distribution due to its robustness.
- 4- Elite children are the individuals with the best fitness values in the current generation who are guaranteed to survive to the next generation. 5 or 10% of population size is considered as elite count. The algorithm is repeated until the number of generations equals 300, which is the termination criterion.
- 5- Heuristic crossover and Gaussian mutation step-size are used to produce offspring for the next generation. Gaussian mutation mean operator is set to zero and the standard deviation  $\delta = 5.5$  in this optimization. Heuristic crossover returns a child with a small distance away from the parents with the better fitness value.

The parameter Ratio can specify how far the child is from the better parent. The following equation illustrates the relation between parameter Ratio and child, as next generation.

Child= parent #2 + R (parent #1- parent #2)

Where parent #1 has the better fitness value than the parent #2 and R is the parameter Ratio. R=1.2 is considered for second order dynamic model.

6- Hybrid function increases the robustness of genetic algorithm, which is run after the genetic algorithm termination in order to improve the value of fitness function. The Hybrid function uses the final point from the genetic algorithm as its initial point for the optimization to converge to the nearest best value that is the global optimum point.

fminsearch is used as the function that is an un-constrained minimization function in the optimization. fminsearch uses the simplex search method of [5]. This direct search method does not use gradients. The results of GA Estimation are collected in Table II.

TABLE II. GENETIC ESTIMATION RESULTS

State No.	Sampling Time (sec.)	Number of Data points	Estimated Parameter k	Estimated Parameter	Gain error %
1	0.0025	20	12.67373	0.3291	0.0046%
2	0.00125	40	12.66903	0.3142	0.0325%
3	0.0005	100	12.66502	0.3124	0.0642%

# V. LSE ESTIMATION

The equation (4) can be written in time domain as follows:

$$\frac{d^{2}y(t)}{dt^{2}} + 2\eta\omega_{n}\frac{dy(t)}{dt} + \omega_{n}^{2}y(t) = k\omega_{n}^{2}x(t)$$
 (7)

Digitization is the first step in LSE modeling, which is carried out by the following differentiation approximation.

$$\frac{dy(t)}{dt} = \frac{y(t) - y(t - T_s)}{T_s} \quad for \quad T_S << 1$$
 (8)

In general form, any other approximations can be used. The digitized model can be written as:

$$y(t) - 2y(t - T_s) + y(t - 2T_s) + \omega_n^2 T_s^2 y(t - 2T_s) + 2\eta \omega_n T_s \left[ y(t - T_s) - y(t - 2T_s) \right] = k\omega_n^2 T_s^2 x(t - 2T_s)$$
(9)

 $\omega_n T_s = \alpha$ , so the model structure can be written as follows:

$$\Psi(t) = \varphi^{T}(t)\theta \tag{10}$$

$$\Psi(t) = y(t) - 2y(t - T) + y(t - 2T) + \alpha^2 y(t - 2T)$$
(11)

$$\Psi(t)$$
 is measurable quantity. (12)

$$\varphi^{T}(t) = \left[2\alpha(y(t-2T) - y(t-T)) \quad \alpha^{2}x(t-2T)\right]$$
(13)

$$\theta(t) = \begin{bmatrix} \eta \\ k \end{bmatrix} \tag{14}$$

 $\theta(t)$  is a two vector of unknown parameters.

$$\hat{\theta} = \left[\sum_{t=1}^{N} \varphi(t) \varphi^{T}(t)\right]^{-1} \left[\sum_{t=1}^{N} \varphi(t) \Psi(t)\right]$$
(15)

After using gradient base LSE [1],  $\eta$  and k will be obtained. The results of LSE Estimation are presented in Table III.

#### VI. PRACTICAL EXPERIMENTAL SETUP

The experimental setup consists of a servomotor with 1 KW rating that is connected to the load via a precision 1:30 gearbox, as in Figure 5. An incremental Encoder is coupled to the Servo output to measure the speed of the servo system.

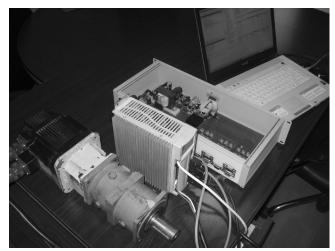


Figure 5. Experimental setup.

The speed output is fed to the servo driver as an internal feedback to make the system as linear as possible. The data acquisition system is a rack-mount controller system that is FPGA based and it is supervised from the PC via USB2 serial port connection, as in Figure 6. Computer is used in the system to receive commands and set points from the user and apply them to the servo setup. The Computer has the authority to stop the process, the data is gathered in the computer, the GA process algorithm is run and the results are saved as the system setup. The experimental results are shown in the next section.

$T_{\ell}$	ABLE	III.	FIRST	ESTIM	IATION	Resu	LTS

TABLE III. FIRST ESTIMATION RESULTS					
State No.	Sampling Time (sec.)	Number of Data points	Estimated Parameter k	Estimated Parameter	Gain error %%
1	0.0001	500	12.673	0.3244	0.0012 %
2	0.00005	1000	12.673	0.3174	4.0489e-4 %
3	0.00001	5000	12.673	0.3117	4.0489e-4 %



Figure 7. GUI Software.

In the experimental setup, the estimated parameters are used for the overall control, and the transients during startup are used instead of persistence excitation of the input. With respect to the computer, there is a complicated process going on, as it has been mentioned in the previous sections. The GUI software is illustrated in fig. 7.

# VII. GA-ESTIMATION IN COMPARISON WITH LSE-ESTIMATION

The difference between GA Estimation and LSE Estimation in servo start up will be remarkable when the number of data points decreases. The results of practical experimentation illustrate that high resolution in estimation using LSE method will be obtained since increasing data point numbers and increasing sampling frequency with same data point numbers, the robustness of GA Estimation can be compared with LSE Estimation. Figures 8, 9, 10 and 11 indicate the resolution of estimation using GA & LSE methods with N=20, N=40.

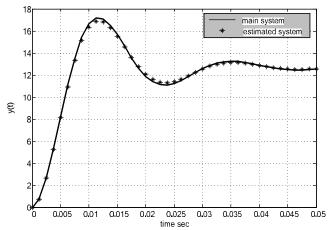
Tables IV and V show the resolution of each method.

TABLE IV. COMPARISON BETWEEN GA & LSE METHODS FOR N = 20, 40

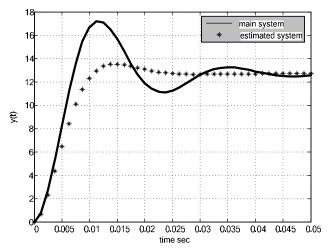
Estimation method	Sampling Time (sec.) T <sub>s</sub>	Number of Data points N	Estimated Parameter k
1-GA	0.0025	20	12.67373
2-LSE	0.0023	20	12.6835
1-GA	0.00125	40	12.66903
2-LSE	0.00123		12.6732

TABLE V. RESOLUTION COMPARISON OF TWO METHODS FOR N = 20, 40

Estimation method	Number of Data points N	Estimated Parameter $\eta$	Resolution Percentage
1-GA	20	0.3291	93.94 %
2-LSE	20	0.6540	10.76 %
1-GA	40	0.3142	98.74 %
2-LSE	40	0.4825	44.50 %



**Figure 8.** GA Estimation for N = 20 (resolution = 93.94%).



**Figure 9.** LSE Estimation for N = 20 (resolution = 10.76%).

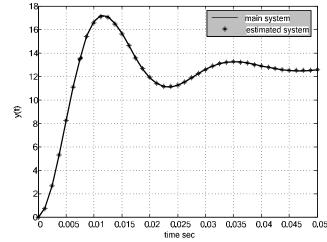
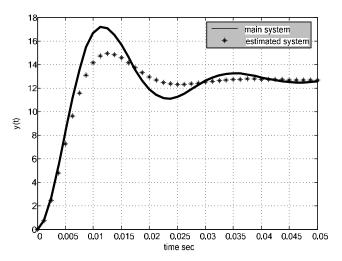


Figure 10. GA Estimation for N = 40 (resolution = 98.74%).



**Figure 11.** LSE Estimation for N = 40 (resolution = 44.50%).

## VIII. CONCLUSION

The dynamic model of closed loop Servo was described using two methods: LSE estimation and GA estimation. This proposed method is applicable in off line parameter estimation, according to robustness of GA optimization in finding global optimum in nonlinear models with respect to estimated parameters. The GA method is able to estimate parameters in high resolution.

Since the number of data points in GA estimation algorithm is smaller than the number of data points in LSE estimation, this method provides accurate parameter estimates. Accurate parameter estimation is satisfied by this method, especially the estimation of parameter  $\eta$  which is important in identifying the model dynamic state.

Despite LSE estimation, GA estimation can be used for systems that are not linear due to parameters.

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