New Code Matched Interleaver for Turbo Codes with Short Frames

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***Abstract*—Turbo codes are a parallel concatenation of two or more convolutional codes, separated by interleavers, therefore their performance is not influenced just by the constituent encoders, but also by the interleaver. For short frame turbo codes, the selection of a proper interleaver becomes critical. This paper presents a new algorithm of obtaining a code matched interleaver leading to a very high minimum distance and improved performance.**

***Index Terms*—communication standards, information theory, interleaved coding, modulation coding, permutation codes**

I. INTRODUCTION

Turbo codes behave very well at a low signal to noise ratio (SNR) because of their sparse distance spectrum, thus generating a low multiplicity of low-weight code-words and a large multiplicity of average weight code-words. This phenomenon is known as spectral thinning [1]. The sparse distance spectrum determines the error correction performance to be influenced at low SNR by the large number of medium weight code-words, whereas at medium to high SNR, the bit error rate (BER) and frame error rate (FER) curves are determined by the few low weight code-words. In this situation, turbo codes experience an error-floor limitation, because of their low minimum distance [2]. The BER and FER curves are bounded by the following union bounds shown in equations (1) and (2).

$$FER \le 0.5 * \sum_{d=d_{free}}^{L/R} n_d erfc\left(\sqrt{\frac{dRE_b}{N_0}}\right) \tag{1}$$

$$BER \le 0.5 * \sum_{d=d_{free}}^{L/R} \frac{\omega_d}{L} erfc\left(\sqrt{\frac{dRE_b}{N_0}}\right) \tag{2}$$

where L is the length of the interleaver; R is the coding rate; n_d is the number of code-words that have a Hamming weight equal to d; ω_d is the total weight of all the information sequences that generate code-words of weight d; erfc is the error function complement; N_0 is the one-sided noise density; E_b is the bit energy.

At high and moderate SNRs, the error floor can be estimated through the bounds from equations (3) and (4).

$$FER \approx 0.5 * N_{free} * erfc\left(\sqrt{d_{free} * \frac{E_b}{N_0} * R}\right) \tag{3}$$

$$BER \approx 0.5 * \frac{\omega_{free}}{L} * erfc\left(\sqrt{d_{free} * \frac{E_b}{N_0} * R}\right) \tag{4}$$

where d_{free} is the free distance; N_{free} is the multiplicity of the code words with the Hamming weight equal to d_{free} and ω_{free} is the sum of the N_{free} information words that produce code words with the Hamming weight equal to d_{free}.

In order to lower the error floor, the increase of the free distance is mandatory. This can be accomplished either through the increase of the size of the interleaver, or through proper interleaver design. Increasing the interleaver size leads to longer delays and larger memory requirements, fact that is intolerable in standards such as Digital Video Broadcasting (DVB) or Universal Mobile Telecommunication Standard (UMTS). Furthermore, some mobile radio systems have short frames, typically under 300 bits, but require a high Quality of Service (Qos)[3]. For the above mentioned situations the best option is to use a code matched interleaver [4], [5].

II. THE SYSTEM MODEL

The turbo encoder used in the simulation is symmetrical and uses two identical convolutional encoders with the feed-forward and feed-back polynomials expressed in octal as 15 and 13, respectively. In Figure 1 the structure of the encoder is depicted. X_k is the systematic output from the first encoder, Z_k and Z'_k are the parity outputs of the first and second convolutional encoders. The systematic output from the second encoder is punctured [6].

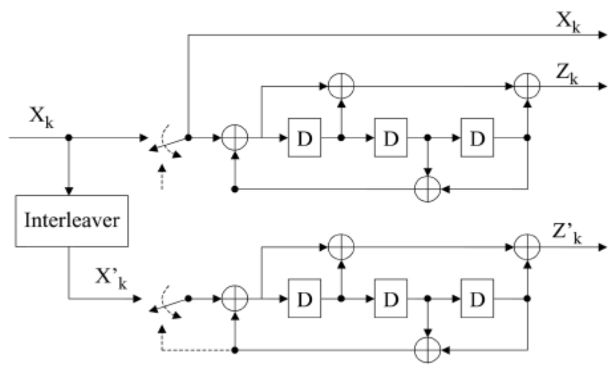

Figure 1. The structure of the turbo encoder.

The turbo encoder has a post-interleaver trellis termination (flushing), which means that both convolutional encoders are reset in an independent manner. This is done by commuting the two switches from the on state (after a number of clock cycles equal to the size of the interleaver) to the off state (for a number of three clock cycles, which is equal to the memory of the constituent convolutional encoders).

This kind of termination technique offers superior results to no trellis termination at all, but when compared to the

Digital Object Identifier 10.4316/AECE.2010.01019

dual trellis termination method is still vulnerable to interleaver edge effects (due to the fact that the tail bits are not interleaved, low weight code-words can be generated when a weight one input sequence with the one bit near the end of one constituent code maps to the near end position in the other constituent code) [7]. The main advantage of this type of termination strategy over dual termination is the increased code rate, which is essential especially when considering communication protocols with short frames. The code rate for UMTS with post-interleaver termination is greater than the code rate for dual termination using the same component encoder, as shown in equation (5). The inequality (5) is analyzed considering the fact that the UMTS frame has a minimum length of 40 and the memory of the encoders is 3.

$$R_{C\,dual} = \frac{L-6}{12L} \le R_{Cpost} = \frac{L}{3L+12} \tag{5}$$

When a non-binary modulation is used, the coding gain for the fading channel can be increased by using BICM (Bit Interleaved Coded Modulation) [8]. Standard coding-modulation schemes, such as TCM (Trellis Coded Modulation), are optimum for the AWGN channel, in the sense that they maximize the free Euclidian distance between code-words. In case of an independent Rayleigh channel, the performance of the coding-modulation scheme is not influenced by the Euclidian distance, but rather by the Hamming free distance, which is not maximized by TCM. BICM on the other hand, separates coding from modulation and hence cannot achieve optimum Euclidian distance. However, it can achieve a free Hamming distance larger than TCM. Through BICM, the non-binary channel generated by the multi-level constellation of size 2^m is transformed into m parallel independent binary channels. The independence between the parallel channels is ensured by the bitwise interleaver [9].

In the simulation scenarios, a random bitwise interleaver is used between the encoder and the BPSK (Binary Phase Shift Keying) modulator, as shown in Figure 2. Because BPSK is a binary modulation, the Hamming and the Euclidean distances are proportional, so an increase of the Hamming distance results in an increase of the Euclidean distance as well [10]. Additionally, BICM enables the use of an iterative demodulation-decoding technique (BICM-ID), which further enhances its performance [11].

Figure 2. The Bit Interleaved Coded Modulation Principle

Two sets of simulations are run, supposing that the channel is either an AWGN (Additive White Gaussian Noise) channel, or a Rayleigh Multiplicative Fading (RMF) channel. The turbo decoder used is based on a SW-SISO (sliding window-soft input soft output) iterative algorithm with two MAP (Maximum A Posteriori) decoders implemented in the log-domain [12].

III. Generic Unmatched Interleavers

A. Block interleaver:

This type of interleaver formats the data frame of length K into a matrix with N rows and M columns, with K=N*M. The data is written row-wise and the reading is performed column-wise. The structure of this interleaver is given in equation (6):

$$\pi\left(i + j \cdot M + 1\right) = i \cdot N + j + 1 \tag{6}$$

With the restrictions:

$$i \in \{0,1,\dots,M-1\};\, j \in \{0,1,\dots,N-1\} \tag{7}$$

B. Random interleaver:

This interleaver is constructed by generating a random dither vector of length K. The permutation is given by ascending or descending sorting of the dither vector.

C. Welch-Costas interleaver:

This interleaver is described by the Costas permutation given in equation (8):

$$\pi(i) = a^i \bmod (K+1),\ 0 \le i < K \tag{8}$$

where the length of the interleaver is K=p-1, with p a prime number and a is a primitive element, which has the property that $\{1, a, a^2, \dots, a^{p-2}\}$ modulo p are distinct.

D. S-Random interleaver:

This kind of interleaver is randomly generated and its elements respect an user imposed S spreading factor. The algorithm is initialized by generating an empty vector of size K. The element i of the vector is randomly chosen in order to comply to differ from the last S elements with a value of at least S, thus satisfying the previously mentioned condition. The generation time is reasonable, provided that the spreading value S is less than $\sqrt{K/2}$ [13]. The spreading factor S is defined by the equation (9):

$$|i-j| \le S \Rightarrow |\pi(i) - \pi(j)| > S;\ (i;j) \in [0; K-1] \tag{9}$$

IV. The Proposed Code Matched Interleaver

The main idea behind the proposed code matched interleaver design is to improve the last three spectral lines. In order to generate a high performance code-matched permutation, a method of computing the distance spectrum of the specific code is necessary. Furthermore, because in the simulations, the post-interleaver trellis termination is considered, the distance spectrum calculation algorithm has to take this aspect into account as well. There are several distance measurements methods, such as the true distance measurement method [14], the error-impulse method [15], the all-iterative decoding method [16] or the double impulse iterative decoding method [17]. From all of these, the most reliable is the true distance measurement method, which is able to reliably compute the first three terms of the distance spectrum. The disadvantage of this approach is that the complexity increases severely with the free distance (which in its turn is dependent on the interleaver's length).

The design algorithm for the code matched interleaver can be synthesized as follows:

1). Start from a given interleaver. The cases studied in this paper are the Block, the Random, the Welch-Costas and the S-Random interleavers.

2). Calculate the S-spread of the interleaver and the first three terms of the distance spectrum {d(1), n(1), w(1)},{d(2), n(2), w(2)} and {d(3), n(3), w(3)} taking into consideration the post-interleaver flushing termination. Furthermore, the normalized dispersion γ and the S_{new} spreading factor are computed. A cost function I is defined, where $I=(S+S_{new})*\gamma$

3). For the desired number of iterations perform the following operations:

4). With two indexes i and j that are incrementally built, the interleaver can be completely scanned, and the positions given by these indexes are swapped

5). The swap is kept only if a series of conditions in the following order are met:

a). If the new interleaver doesn't have a spreading factor at least equal to the initial S-spread value, the swap is discarded and the algorithm returns to step 4, otherwise jump to 5.b

b). The first term of the distance spectrum is computed. If there is an improvement in the sequence d(1)-n(1)-w(1) (if FER optimization is desired) or in the sequence d(1)-w(1)-n(1) (if BER optimization is desired) the swap is kept and the algorithm returns to step 4. In case there is no change in the distance spectrum, the algorithm computes the second term of the distance spectrum and makes the same evaluation, keeping only the swap that improves the second term. If still there is no improvement in the distance spectrum the third term is computed and the same procedure is applied. The swap is kept if there is an improvement and discarded if the distance spectrum is damaged. In case there is no change in the distance spectrum after computing the first three terms, then the algorithm jumps to step 5.c

c). The normalized dispersion γ, the S_{new} spreading factor and the cost function $I=(S+S_{new})*\gamma$ are computed. The swap is kept if the cost function suffers an improvement, otherwise the algorithm jumps to step 4.

The algorithm has several advantages over other code matched interleavers. First of all, there is the possibility to start from any kind of interleaver. The choice of the start interleaver influences the final performance of the code matched interleaver. Provided a very good interleaver is chosen as the start-up structure, then not only the performance would be improved, but also the generation time will be less, because a lower number of iterations would have to be performed. Another advantage of this design is that given by the flexibility, not only in terms of the number of iterations that are user definable, but also from the point of view of FER or BER optimization. Third of all, a real distance spectrum calculation algorithm is used, instead of estimating the distance spectrum with the help of various error patterns. Finally, in case a non-random interleaver is selected as starting interleaver, the design is fully deterministic, thus having the advantage of being easy to implement in VLSI structures, with a low memory capacity.

V. Simulations And Results

Two sets of simulations were performed, considering the channel as being either an AWGN or RMF channel. Code matched interleavers of length 100 (matched-Random and matched-Welch-Costas interleavers) and 160 (matched-Block and matched-S-Random interleavers) were generated, starting from the Random, Welch-Costas, Block and S-Random interleavers. The number of iterations per interleaver was set to 3, the MAP decoder was set to 12 iterations and the modulation used was BPSK with BICM-ID, also set to 12 iterations. The FER curves are shown in figures 3 and 4 for length L1=100 and in figures 5 and 6 for length L2=160. Furthermore, the spectral distances and the most important parameters are shown for all the starting and generated interleavers for the case of post-interleaver termination in tables I, II, III and IV for length L1=100 and tables V, VI, VII and VIII for length L2=160. The results of the simulations yield to a clear improvement of the performance, due to the better distance spectrum and the proper choice of the cost function I, which maximizes the dispersion γ. From tables I-VIII, it can be deducted that the significant higher free distances achieved by the code-matched interleavers, result in fewer summation terms in equations (1) and (2). Furthermore, higher dispersion values, lead to lower multiplicities of low-weight code-words in the distance spectrum, which is the case for matched-block and matched-S-Random interleavers.

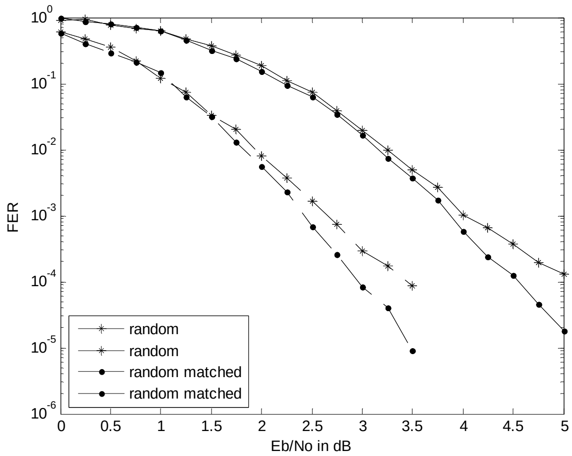

Figure 3. FER for length L1=100 in AWGN and RMF.

TABLE I. Parameters of the Random Interleaver for L1=100

d_{free}	n_{free}	w_{free}	S	S_{new}	γ
12	1	4	0	2	0.81

TABLE II. Parameters of the Matched-Random Interleaver for L1=100

d_{free}	n_{free}	w_{free}	S	S_{new}	γ
23	20	66	1	3	0.81

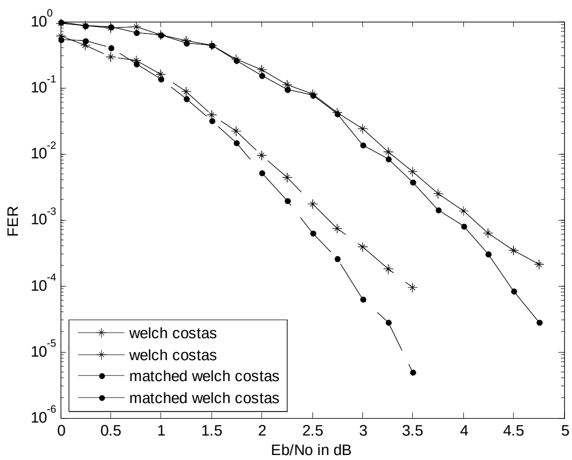

Figure 4. FER for length L1=100 in AWGN and RMF.

TABLE III. PARAMETERS OF THE WELCH-COSTAS INTERLEAVER FOR L1=100

d_{free}	n_{free}	w_{free}	S	S_{new}	γ
12	1	4	0	2	1

TABLE IV. PARAMETERS OF THE MATCHED-WELCH-COSTAS INTERLEAVER FOR L1=100

d_{free}	n_{free}	w_{free}	S	S_{new}	γ
23	20	75	1	3	0.81

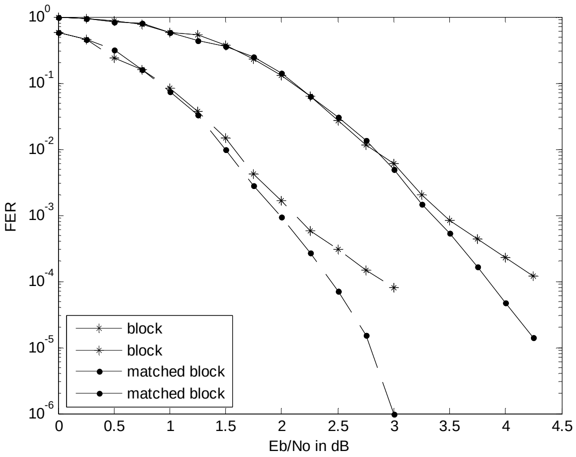

Figure 5. FER for length L2=160 in AWGN and RMF.

TABLE V. PARAMETERS OF THE BLOCK INTERLEAVER FOR L2=160

d_{free}	n_{free}	w_{free}	S	S_{new}	γ
19	2	2	9	11	0.02

TABLE VI. PARAMETERS OF THE MATCHED-BLOCK INTERLEAVER FOR L2=160

d_{free}	n_{free}	w_{free}	S	S_{new}	γ
23	2	4	9	11	0.48

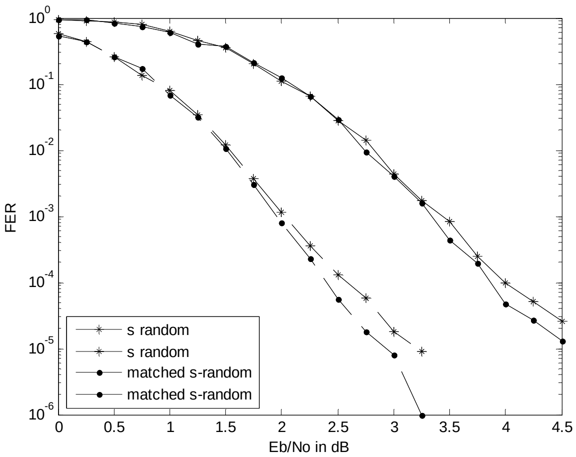

Figure 6. FER for length L2=160 in AWGN and RMF.

TABLE VII. PARAMETERS OF THE S-RANDOM INTERLEAVER FOR L2=160

d_{free}	n_{free}	w_{free}	S	S_{new}	γ
18	5	10	9	11	0.76

TABLE VIII. PARAMETERS OF THE MATCHED-S-RANDOM INTERLEAVER FOR L2=160

d_{free}	n_{free}	w_{free}	S	S_{new}	γ
22	1	2	9	11	0.79

VI. CONCLUSION AND FUTURE WORK

This paper presents a new code matched interleaver design. Its performances are evaluated against various starting interleavers, in case of both AWGN and RMF channels, for two different short length frame sizes. The results of the simulations show a clear improvement in all considered scenarios. Among the advantages of this new design algorithm are flexibility, ease of implementation and increased performance. Future work should deal with the study of the code matched interleaver for longer frame sizes and the comparative performance against some of the best known interleaver types such as Almost Regular Permutation (ARP), Quadratic Polynomial Permutation (QPP) and Dithered Relative Prime (DRP).

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