# Low Complexity Hybrid Precoding for Broadband mmWave Massive MIMO Systems

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***Abstract*—Hybrid precoding becomes a candidate for Millimeter wave (mmWave) massive MIMO (Multiple-Input and Multiple-Output) systems because it can extremely reduce power consumption and high costs. Most prior work considered hybrid precoding for narrowband systems. However, wideband systems with frequency selectivity are likely to be operated in the future. In broadband systems, a common analogue precoder is designed for the overall frequency band whereas different digital precoders are employed in different subcarriers. In this paper, we propose the hybrid precoding schemes for broadband mmWave massive MIMO systems. First, the hybrid single-user (SU) algorithm is proposed for a single-user system. The common analogue precoding matrix is derived from the Equal Gain Transmission (EGT) method and the digital precoding matrices for different subcarriers are employed based on directly water-filling technique. Second, the hybrid multi-user (MU) algorithm is proposed for a multi-user system. Gram-Schmidt orthogonalization is added in the analogue domain and zero-forcing (ZF) is utilized for digital precoders in order to nullify inter-user interference. Simulation results show that our proposed hybrid schemes with low complexity can almost reach the performance of fully digital precoding algorithm and outperform other hybrid algorithms.**

***Index Terms*—millimeter wave communication, MIMO, signal processing, wireless communication, wideband.**

## I. Introduction

WITH the development of fifth generation of mobile telecommunications (5G), the peak theoretical transmission speed can reach up to 40 Gbit per second. The wide bandwidths available at the mmWave frequencies cause mmWave communication to become a promising candidate for 5G systems. Large antenna arrays deployed at base stations (BS's) are required in order to achieve high quality communication links in mmWave systems. Precoding technology is used at BS's to generate transmitting signals for multiplexing data streams [1]. For traditional lower frequency systems, digital precoding in the baseband is usually applied. However, each antenna requires a lot of high energy-intensive radio frequency (RF) chains for conventional digital precoding. Each RF chain which includes up converter, digital-to-analog converter, *etc* can consume about 250 mW [2]. That is a large portion of the total energy consumption in mmWave massive MIMO systems. If the traditional digital precoding is utilized in mmWave massive MIMO systems, the corresponding large number of RF chains will result in high energy consumption. Therefore, based on current research, precoding for mmWave massive MIMO systems is trend to be divided into analogue and digital domains. The hybrid precoding utilizing the channel sparsity which contains analogue and digital precoding for single-user systems is investigated in [3]. It was shown that hybrid precoding can reduce the number of RF chains required and achieve almost the performance of the fully digital precoding. Further, several hybrid precoding algorithms have been proposed in multi-user scenarios in [4],[5]. The hybrid algorithms proposed in [3]–[5] can approach the performance of full digital precoding with fewer RF chains in flat fading channels with single-subcarrier.

Most prior work of hybrid precoding is confined to the narrow flat fading channels. As mmWave communication is expected to operate on wideband frequency-selective channels, it is important to design hybrid precoding for broadband systems. The main challenge in developing hybrid precoding for broadband systems is how to design a common analogue precoder on all the subcarriers whereas using different digital precoders across each subcarrier. Because the analogue precoding matrix acts as an RF phase shifter, it can provide a constant phase shift response over wideband frequencies. The analogue precoder is assumed to be frequency flat. However, the digital precoders on different subcarriers can be different, which is the most important difference between hybrid precoding in broadband channels and that in narrowband channels. The scheme in [6] aimed to design hybrid precoding with wideband channels for OFDM-based systems. In that paper, hybrid precoding is developed for both single-user and multi-user scenarios. It provided an iterative precoding scheme to reduce the number of RF chains. But the iterative algorithm relied on the exhaustive search has high complexity. The main idea in [7] which has low complexity was designed for limited feedback frequency selective systems based on Gram-Schmidt orthogonalization. The optimal baseband precoders are firstly derived as functions of the analogue precoders. Then, the greedy hybrid precoding algorithm is designed. The main limitation of [7] is that it still requires eigenvalue calculation and exhaustive search.

In this paper, we propose hybrid precoding schemes for SU (single-user) and MU (multi-user) in broadband

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systems. In the SU scenario, the hybrid SU algorithm is proposed. The common analogue precoding matrix is derived from the EGT method [8] and water-filling [9] is directly utilized in the digital domain for SU systems that avoids the requirement for singular value decomposition (SVD) based on the effective channel. In the MU scenario, the hybrid MU algorithm is proposed. Gram-Schmidt orthogonalization [10] is applied in the analogue domain. And zero-forcing [11] is applied in the digital domain in order to greatly reduce the interference among different users. Perfect channel state information (CSI) is assumed at the BS. Channel estimating methods are out of the scope in this paper, though [1], [12] designed some channel estimation techniques for mmWave massive MIMO systems. Simulation results show that the algorithms approach to the full digital performance and get a better performance than other algorithms. Moreover, the proposed algorithms have low computation complexity.

The contribution of this paper is as follows.

- We investigate hybrid precoding for broadband systems with frequency-selective channels, while most prior work is confined to the narrow flat fading channels. The main challenge for broadband systems is how to design a common analogue precoder on all the subcarriers whereas different digital precoders are utilized across each subcarrier.
- In the single-user scenario, the SU algorithm with low-complexity is developed. The common analogue precoder is derived from the EGT. Because the effective channel is almost orthogonal according to *Proposition 1*, water-filling method can be directly utilized in the digital precoder without singular value decomposition. This method can reduce the computational complexity of the algorithm since there is no requirement for singular value decomposition (SVD) to obtain parallel sub-channels.
- In the multi-user scenario, the MU algorithm with low-complexity is developed. The analogue precoder of MU system is derived from the EGT with the addition of a Gram-Schmidt orthogonalization process. This is based on the intuition that the analogue precoding vector of each column is better to be orthogonal (or nearly orthogonal) according to *Remark 1*. The analogue precoder is first designed by maximizing sum rate. Then the conventional digital precoding scheme is applied to remove the inter-user interference.

The remaining of this paper is organized as follows. Section 2 and 3 introduce the system models for SU and MU and formulate the sum-rate maximization problem. The proposed precoding algorithms design and complexity analysis are drawn in Section 4. In Section 5, simulations are carried out to evaluate the performance of the proposed schemes. Conclusions are presented in Section 6.

*Notation:* In this paper, $\mathrm{E}(\cdot)$ denotes the expectation; boldface letters denote vectors and matrices; $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$ and $(\cdot)^{-1}$ denote the transpose, conjugate transpose, and inversion, respectively; $|\cdot|$ denotes the determinant of a matrix; $\|\cdot\|_{\mathrm{F}}$ denotes the Frobenius norm of a matrix; $\mathbf{I}_{N_r}$ is the $N_{\mathrm{r}} \times N_{\mathrm{r}}$ identity matrix; Finally, $\mathbb{C}^{M\times M}$ represent spaces of $M \times M$ matrices with complex entries.

## II. SYSTEM MODEL FOR SINGLE-USER

### A. SIGNAL MODEL FOR SU

The design of hybrid single precoding problem for the broadband frequency-selective channel is considered firstly. One BS equipped with $N_{\mathrm{BS}}$ antennas serves one user by sending $N_{\mathrm{S}}$ data symbols on different subcarriers. One MS is equipped with $N_{\mathrm{r}}$ antennas. In this paper, it is assumed subcarriers for simplicity's sake that an equal number of data streams for all subcarriers, since all the sub-channels are low rank with highly correlated channels for mmWave systems. In addition, the number of RF chains $N_{\mathrm{RF}}$ cannot exceed the number of transmitter and receiver antennas, i.e. $N_{\mathrm{RF}} \le \min\left(N_{\mathrm{BS}}, N_{\mathrm{r}}\right)$ . A broadband mmWave massive MIMO system model for SU is shown in Fig. 1. The hybrid precoding architecture for SU is adopted, which includes a high-dimensional analogue precoder and a low-dimensional digital precoder.

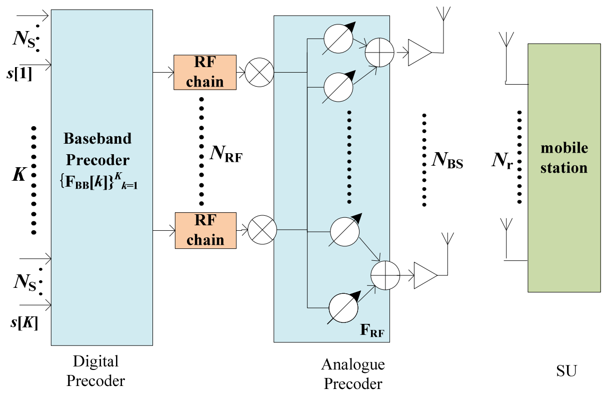


Figure 1. A broadband mmWave massive MIMO system model for SU

Firstly, the BS transmits $N_{\mathrm{S}}$ data symbols $\mathbf{s}[k]$ at each subcarrier $k=[1,2,\ldots,K]$ . Digital precoder is a low-dimensional matrix, $\mathbf{F}_{\mathbf{BB}}[k] \in \mathbb{C}^{N_{\mathrm{RF}}\times N_{\mathrm{S}}}$ , followed by an analogue precoder, $\mathbf{F}_{\mathbf{RF}} \in \mathbb{C}^{N_{\mathrm{BS}}\times N_{\mathrm{RF}}}$ . The RF precoding matrix $\mathbf{F}_{\mathbf{RF}}$ is identical for all subcarriers. The sampled transmitting signal at subcarrier *k* is therefore

$$\mathbf{x}[k] = \mathbf{F}_{\mathbf{RF}}\mathbf{F}_{\mathbf{BB}}[k]\mathbf{s}[k] \tag{1}$$

where $\mathbf{s}[k]$ is a $N_{\mathrm{S}} \times 1$ vector, which is the message to be transmitted at subcarrier *k*, and normalized as $\mathrm{E}\left[\mathbf{s}[k]\mathbf{s}[k]^{\mathrm{H}}\right] = \left(1/KN_{\mathrm{S}}\right)\mathbf{I}_{N_{\mathrm{S}}}$. Since $\mathbf{F}_{\mathbf{RF}}$ is implemented as an analogue phase shifter and the entries of $\mathbf{F}_{\mathbf{RF}}$ are constant modulus whose entries should be normalized as $\left|\mathbf{F}_{\mathbf{RF}}[i,j]\right| = 1/\sqrt{N_{\mathrm{BS}}}$, $\forall i = 1,2,\ldots,N_{\mathrm{BS}}$, $j = 1,2,\ldots,N_{\mathrm{RF}}$, where $\mathbf{F}_{\mathbf{RF}}[i,j]$ is the $(i,j)^{th}$ element of $\mathbf{F}_{\mathbf{RF}}$. A total power constraint of the hybrid precoder is enforced by $\sum_{k=1}^{K}\left\|\mathbf{F}_{\mathbf{RF}}\mathbf{F}_{\mathbf{BB}}[k]\right\|_{\mathrm{F}}^{2} = KN_{\mathrm{S}}$.

A broadband channel model is adopted, in which the received signal observed at subcarrier *k* by the MS is

$$\mathbf{y}[k] = \sqrt{\rho}\mathbf{H}[k]\mathbf{F}_{\mathbf{RF}}\mathbf{F}_{\mathbf{BB}}[k]\mathbf{s}[k] + \mathbf{n}[k] \tag{2}$$

where $\mathbf{H}[k]$ is an $N_r \times N_{BS}$ matrix that indicates the channel at subcarrier $k$ between the BS and the user, $\mathbf{n}[k] \in \mathcal{CN}(0, \sigma^2)$ represents the complex Gaussian noise at subcarrier $k$, and $\rho$ represents the average received power.

### B. Channel Model for SU

The broadband mmWave channel is different from the conventional rich scattering propagation environment. Due to limited scattering and broadband characteristics, a geometric broadband mmWave channel model [13] is adopted in this paper. In this model, a geometric channel has $N_c$ clusters and $L_u$ paths in each cluster, where the channel matrix for subcarrier $k$ is written as

$$\mathbf{H}[k] = \sqrt{\frac{N_{BS} N_r}{N_c L_u}} \sum_{c=1}^{N_c} \sum_{l=1}^{L_u} \alpha_{cl} \mathbf{a}_{\mathbf{MS}}(\theta_{cl}) \mathbf{a}_{\mathbf{BS}}^{H}(\phi_{cl}) e^{-j2\pi\varphi_c \frac{k}{K}} \quad (3)$$

where $\alpha_{cl} \in \mathcal{CN}(0, N_{BS} N_r / N_c L_u)$ is the complex gain of the $l^{th}$ path in $c^{th}$ cluster which includes the path loss. The variable $\theta_{cl} \in [0, 2\pi]$ is the $l^{th}$ path's angle of arrival (AoA). The variable $\phi_{cl} \in [0, 2\pi]$ is the $l^{th}$ path's angle of departure (AoD). Further, $\mathbf{a}_{\mathbf{MS}}(\theta_{cl})$ is the antenna array response vector of the MS, $\mathbf{a}_{\mathbf{BS}}(\phi_{cl})$ is the antenna array response vector of the BS. $N_c$ is the number of clusters in the channel with $c \in [1, 2, \ldots, N_c]$. $L_u$ is the number of propagation paths in $c^{th}$ cluster with $l \in [1, 2, \ldots, L_u]$. At last, $\varphi_c$ is proportional to the phase shift in $c^{th}$ cluster. A uniform linear array (ULA) is applied in our simulations. If a ULA is equipped, $\mathbf{a}_{\mathbf{BS}}(\phi_{cl})$ can be defined as

$$\mathbf{a}_{\mathbf{BS}}(\phi_{cl}) = \frac{1}{\sqrt{N_{BS}}} \left[1, e^{j\frac{2\pi}{\lambda} d \sin(\phi_{cl})}, \ldots, e^{j(N_{BS}-1)\frac{2\pi}{\lambda} d \sin(\phi_{cl})}\right]^T \quad (4)$$

where $\lambda$ represents the wavelength of mmWave and $d$ represents the distance between adjacent antenna elements. $\mathbf{a}_{\mathbf{MS}}(\theta_{cl})$ of the MS can be depicted in a similar pattern. Because we assume that the frequency of mmWave is much larger than total communication bandwidth, the wavelength of signal in each subcarrier can be considered approximately equal.

The channel $\mathbf{H}[k]$ can be described in the following compact form as

$$\mathbf{H}[k] = \mathbf{A}_{\mathbf{MS}} \mathbf{D}[k] \mathbf{A}_{\mathbf{BS}}^{H} \quad (5)$$

where

$$\mathbf{A}_{\mathbf{MS}} = \left[\mathbf{a}_{\mathbf{MS}}(\theta_{11}), \mathbf{a}_{\mathbf{MS}}(\theta_{12}) \ldots, \mathbf{a}_{\mathbf{MS}}(\theta_{N_c L_u})\right]_{N_r \times N_c L_u}$$

$$\mathbf{A}_{\mathbf{BS}} = \left[\mathbf{a}_{\mathbf{BS}}(\phi_{11}), \mathbf{a}_{\mathbf{BS}}(\phi_{12}) \ldots, \mathbf{a}_{\mathbf{BS}}(\phi_{N_c L_u})\right]_{N_{BS} \times N_c L_u}$$

$$\mathbf{D}[k] = diag\begin{pmatrix} \sqrt{\frac{N_{BS} N_r}{N_c L_u}} \alpha_{11} e^{-j2\pi\varphi_c \frac{k}{K}}, \cdots \\ \cdots, \sqrt{\frac{N_{BS} N_r}{N_c L_u}} \alpha_{N_c L_u} e^{-j2\pi\varphi_c \frac{k}{K}} \end{pmatrix}_{N_c L_u \times N_c L_u}$$

### C. Problem Formulation for SU

The main objective for SU scenario is to maximize the overall rate of the system by efficiently designing hybrid analogue and digital precoders at the BS. The BS is assumed to acquire the perfect CSI. According to the received signal captured at subcarrier $k$ in (2), the achievable rate of the MS at subcarrier $k$ is

$$R[k] = \log_2 \left| \mathbf{I}_{N_r} + \frac{1}{\sigma^2} \mathbf{H}[k] \mathbf{F}_{\mathbf{RF}} \mathbf{F}_{\mathbf{BB}}[k] \mathbf{F}_{\mathbf{BB}}^{H}[k] \mathbf{F}_{\mathbf{RF}}^{H} \mathbf{H}[k]^{H} \right| \quad (6)$$

Although other criterias such as the max-min fairness criterion is of interest in [14] or the maximization of energy efficiency(EE) of the system is applied in [15], we still aim to maximize the achievable sum rate in this paper, because we mainly focus on the sum-rate compared to others. The precoder optimization problem can be formulated as

$$\max_{\mathbf{F}_{\mathbf{RF}}, \mathbf{F}_{\mathbf{BB}}[k]_{k=1}^{K}} \frac{1}{K} \sum_{k=1}^{K} R[k] \quad (7a)$$

$$\text{s.t.} \quad \mathbf{F}_{\mathbf{RF}} \in \mathbf{W} \quad (7b)$$

$$\sum_{k=1}^{K} \left\| \mathbf{F}_{\mathbf{RF}} \mathbf{F}_{\mathbf{BB}}[k] \right\|_F^2 = K N_S \quad (7c)$$

Where $\mathbf{W}$ is the feasible domain of the analogue precoders, which is the set of $N_{BS} \times N_{RF}$ matrices with constant-modulus entries $\left(\left|\mathbf{F}_{\mathbf{RF}}[i,j]\right| = 1/\sqrt{N_{BS}}\right)$, $\forall i = 1, 2, \ldots, N_{BS}$, $j = 1, 2, \ldots, N_{RF}$. The total power limitation of the BS is performed by normalizing $\mathbf{F}_{\mathbf{BB}}[k]$ as $\sum_{k=1}^{K} \left\| \mathbf{F}_{\mathbf{RF}} \mathbf{F}_{\mathbf{BB}}[k] \right\|_F^2 = K N_S$. There is no other related hardware constraints imposed on digital precoders.

The optimal solution to (7) is almost impossible to obtain, even without the RF restrictions. However, a near optimal solution can be found through an iterative algorithm, which requires a great deal of training process and feedback overhead. Therefore, the direct solution to the sum-rate maximization problem in (7) is neither easy nor practical. In view of the difficulties of previous precoding algorithms in mmWave and the particularity of the broadband system, we propose hybrid solutions for SU and MU in Section 4.

## III. System Model for Multi-User

### A. Signal Model for MU

The model of hybrid precoder for MU system is now considered, in which one BS with $N_{RF}$ RF chains and $N_{BS}$ antennas communicates with $U$ users simultaneously. Each MS of one user is equipped with only one antenna. The BS communicates with each MS only through one stream. So the total number of transmitting streams is $N_S = U$. In addition, the number of MS's simultaneously served by the BS cannot exceed the number of BS RF chains [16], i.e. $N_S \leq N_{RF}$. The spatial multiplexing gain of this hybrid system for multi-user is limited by $N_{BS} \geq N_{RF}$. For simplicity, we assume a general scenario where the BS serves $U$ MS's simultaneously using $N_{RF} = U$ RF chains [17]. The hybrid precoding architecture for MU is adopted in this paper, as shown in Fig. 2.

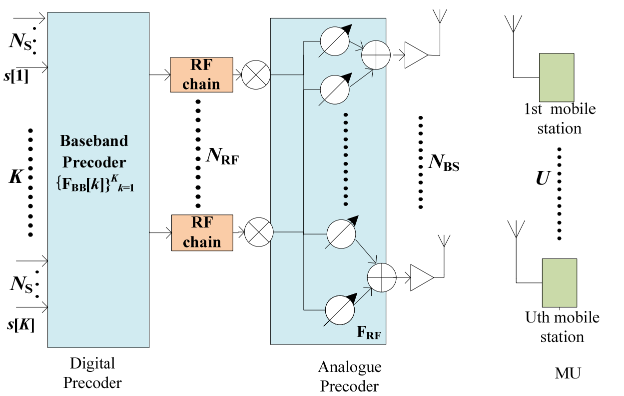


Figure 2. A broadband mmWave massive MIMO system model for MU

On the downlink of muti-user broadband mmWave system, the BS applies $N_{\mathrm{RF}}\times 1$ digital precoders $\mathbf{F}_{\mathbf{BB}}^{u}[k]$ for user $u$ at subcarrier $k$, followed by $N_{\mathrm{BS}}\times N_{\mathrm{RF}}$ analogue precoder. The sampled transmitting signal at subcarrier $k$ is therefore

$$\mathbf{x}[k]=\sum_{u=1}^{U}\mathbf{F}_{\mathbf{RF}}\mathbf{F}_{\mathbf{BB}}^{u}[k]\mathbf{s}_{u}[k] \tag{8}$$

where $\mathbf{s}_u\left[k\right]$ is the message to be transmitted, which is a $N_{\mathrm{S}}\times 1$ vector at subcarrier $k$, and normalized as $\mathrm{E}\left[\mathbf{s}_u\left[k\right]\mathbf{s}_u\left[k\right]^{\mathrm{H}}\right]=\left(1/KN_{\mathrm{S}}\right)\mathbf{I}_{N_{\mathrm{S}}}$. $\mathbf{F}_{\mathbf{RF}}$ is constant modulus, which is the same as the definition in SU system. The hybrid precoding matrix is enforced for a total power constraint to satisfy $\sum_{k=1}^{K}\left\|\mathbf{F}_{\mathbf{RF}}\mathbf{F}_{\mathbf{BB}}^{u}[k]\right\|_{\mathrm{F}}^{2}=KN_{\mathrm{S}},\forall u=1,2,\cdots U$. The received signal observed by the $u^{\text{th}}$ MS at subcarrier $k$ is

$$\mathbf{y}_u[k]=\sqrt{\rho}\mathbf{H}_u[k]\sum_{u=1}^{U}\mathbf{F}_{\mathbf{RF}}\mathbf{F}_{\mathbf{BB}}^{u}[k]\mathbf{s}_u[k]+\mathbf{n}_u[k] \tag{9}$$

where $\mathbf{H}_u[k]$ is the $1\times N_{\mathrm{BS}}$ vector that indicates the channel between the BS and the $u^{\text{th}}$ user in the $k^{\text{th}}$ subcarrier.

### B. Channel Model for MU

The channel between the BS and the $u^{\text{th}}$ user in the $k^{\text{th}}$ subcarrier can be expressed as $\mathbf{H}_u[k]$.

$$\mathbf{H}_u[k]=\sqrt{\frac{N_{\mathrm{BS}}N_r}{N_cL_u}}\sum_{c=1}^{N_c}\sum_{l=1}^{L_u}\alpha_{ucl}\mathbf{a}_{u,\mathrm{MS}}\left(\theta_{u,cl}\right)\mathbf{a}_{u,\mathrm{BS}}^{\mathrm{H}}\left(\phi_{u,cl}\right)e^{-j2\pi\varphi_{uc}\frac{k}{K}} \tag{10}$$

where the expression of the antenna array response vector and complex gain are the same as that in the SU scenario. A $U\times N_{\mathrm{BS}}$ matrix $\mathbf{H}\mathrm{U}[k]=\left[\mathbf{H}_1[k]^{\mathrm{T}},\mathbf{H}_2[k]^{\mathrm{T}},\ldots,\mathbf{H}_U[k]^{\mathrm{T}}\right]^{\mathrm{T}}$ is defined, which indicates the channel between the BS and multiple users at subcarrier $k$. If each user has one antenna, no AoAs exist [18]. So (10) can be simplified as

$$\mathbf{H}_u[k]=\sqrt{\frac{N_{\mathrm{BS}}N_r}{N_cL_u}}\sum_{c=1}^{N_c}\sum_{l=1}^{L_u}\alpha_{u,cl}\mathbf{a}_{u,\mathrm{BS}}^{\mathrm{H}}\left(\phi_{u,cl}\right)e^{-j2\pi\varphi_{u,c}\frac{k}{K}} \tag{11}$$

And the channel between the BS and the $u^{\text{th}}$ user $\mathbf{H}_u[k]$ can be written in the following form as

$$\mathbf{H}_u\left[k\right]=\mathbf{d}_u\left[k\right]\mathbf{A}_{u,\mathbf{BS}}^{\mathrm{H}} \tag{12}$$

where

$$\mathbf{A}_{u,\mathbf{BS}}=\left[\mathbf{a}_{u,\mathbf{BS}}\left(\phi_{11}\right),\mathbf{a}_{u,\mathbf{BS}}\left(\phi_{12}\right)\ldots,\mathbf{a}_{u,\mathbf{BS}}\left(\phi_{N_cL_u}\right)\right]_{N_{\mathrm{BS}}\times N_cL_u}$$

$$\mathbf{d}_u[k]=\left(\sqrt{\frac{N_{\mathrm{BS}}N_r}{N_cL_u}}\alpha_{11}e^{-j2\pi\varphi_c\frac{k}{K}},\ldots,\sqrt{\frac{N_{\mathrm{BS}}N_r}{N_cL_u}}\alpha_{N_cL_u}e^{-j2\pi\varphi_c\frac{k}{K}}\right)_{1\times N_cL_u}$$

### C. Problem Formulation for MU

The data rate in $k^{\text{th}}$ subcarrier for the $u^{\text{th}}$ user in (9) can be described as

$$R_u[k]=\log_2\left(1+\frac{\rho\left|\mathbf{H}_u[k]\mathbf{F}_{\mathbf{RF}}\mathbf{F}_{\mathbf{BB}}^{u}[k]\right|^2}{\rho\sum_{n\neq u}\left|\mathbf{H}_u[k]\mathbf{F}_{\mathbf{RF}}\mathbf{F}_{\mathbf{BB}}^{n}[k]\right|^2+\sigma^2}\right) \tag{13}$$

Now, proceeding with the design of $\mathbf{F}_{\mathbf{RF}}$ and $\mathbf{F}_{\mathbf{BB}}^{u}[k]$, the problem of hybrid precoder design in order to maximize the weighted sum rate can be expressed as

$$\max_{\mathbf{F}_{\mathbf{RF}},\mathbf{F}_{\mathbf{BB}}[k]_{k=1}^{K}}\frac{1}{K}\sum_{k=1}^{K}\sum_{u=1}^{U}\beta_uR_u[k] \tag{14a}$$

$$\text{s.t.}\quad \mathbf{F}_{\mathbf{RF}}\in\mathbf{W} \tag{14b}$$

$$\sum_{k=1}^{K}\left\|\mathbf{F}_{\mathbf{RF}}\mathbf{F}_{\mathbf{BB}}^{u}[k]\right\|_{\mathrm{F}}^{2}=KN_{\mathrm{S}},\forall u=1,2,\cdots U \tag{14c}$$

where $\beta_u$ represents the priority of $u^{\text{th}}$ user. $\mathbf{F}_{\mathbf{BB}}\left[k\right]\in\mathrm{C}^{N_{\mathrm{RF}}\times U}$ is defined as the overall digital precoder by $\mathbf{F}_{\mathrm{BB}}\left[k\right]=\left[\mathbf{F}_{\mathrm{BB}}^{1}\left[k\right],\mathbf{F}_{\mathrm{BB}}^{2}\left[k\right],\cdots\mathbf{F}_{\mathrm{BB}}^{U}\left[k\right]\right]$.

The sum rate maximization problem in (14) is different from the problem of SU system. Firstly, inter-user interference does not exist in the SU scenario. But in the MU system, inter-user interference should be considered which is collocated in the sum rate expression. Secondly, different priority corresponding to different users weights different data streams in a MU system, whereas all the streams always have the same priority in a SU system.

## IV. Hybrid Precoding for SU and MU

In this section, we propose the SU and MU precoding algorithms and analyze their computational complexity. The calculation of the hybrid precoding is divided into analogue and digital steps. The anolog precoder is designed in the first step and the digital precoder is designed in the second step.

### A. The Proposed SU Algorithm

In conventional systems for low frenquency, traditional digital precoding can be implemented. But it can not be performed to the hybrid precoder directly due to the constant-magnitude constraint of $\mathbf{F}_{\mathbf{RF}}$. Moreover, it's difficult to optimize the hybrid precoder, since the constant-magnitude causes the feasible region nonconvex [4]. Hence, the analogue and digital precoders are separately designed. The proposed SU algorithm is described as Algorithm 1, which contains an analogue precoder and $k$ digital precoders.

The analogue precoding is designed in the first step. In the analogue step, the effective gain is aimed to be maximized. EGT method [8] with low complexity can be applied to achieve this goal. EGT can only adjust the phase of transmitting signal based on channel coefficients without the magnitude adjustment. Since the analogue precoding matrix is identical of all subcarriers, which is the main challenge of developing the hybrid precoding in broadband mmWave systems compared to narrowband systems. We can use the average of all subcarriers to make the average channel matrix $\mathbf{H}$ which described as

$$\mathbf{H} = \frac{1}{K}\sum_{k=1}^{K}\left(\mathbf{H}[k]\mathbf{H}[k]^{\mathrm{H}}\right) \tag{15}$$

The analogue precoding matrix can be described as

$$\mathbf{F_{RF}}\left[i, j\right] = 1/\sqrt{N_{\mathrm{BS}}}e^{-j\angle h_{j,i}} \tag{16}$$

where, $h_{i,j}$ is the $(i, j)^{th}$ element of average channel matrix $\mathbf{H}$, $\angle h_{j,i}$ is the complex phase of $h_{j,i}$ .The analogue precoder is only phase mapping of $\mathbf{H}$ from $\mathrm{C}^{N_{\mathrm{r}}\times N_{\mathrm{BS}}}$ to $\mathrm{C}^{N_{\mathrm{BS}}\times N_{\mathrm{r}}}$ as

$$\mathbf{F_{RF}} = \frac{1}{\sqrt{N_{\mathrm{BS}}}}\begin{bmatrix} e^{-j\angle h_{1,1}} & \cdots & e^{-j\angle h_{N_{\mathrm{r}},1,}} \\ \vdots & \ddots & \vdots \\ e^{-j\angle h_{1,N_{\mathrm{BS}}}} & \cdots & e^{-j\angle h_{N_{\mathrm{r}},N_{\mathrm{BS}}}} \end{bmatrix} \tag{17}$$

Hence, the effective channel matrix $\hat{\mathbf{H}}$ can be depicted as

$$\begin{aligned}\hat{\mathbf{H}} = \mathbf{H}\cdot\mathbf{F_{RF}} &= \begin{bmatrix} \left|h_{1,1}\right|e^{j\angle h_{1,1}} & \cdots & \left|h_{1,N_{\mathrm{BS}}}\right|e^{j\angle h_{1,N_{\mathrm{BS}}}} \\ \vdots & \ddots & \vdots \\ \left|h_{N_{\mathrm{r}},1}\right|e^{j\angle h_{N_{\mathrm{r}},1}} & \cdots & \left|h_{N_{\mathrm{r}},N_{\mathrm{BS}}}\right|e^{j\angle h_{N_{\mathrm{r}},N_{\mathrm{BS}}}} \end{bmatrix} \\ &\times\frac{1}{\sqrt{N_{\mathrm{BS}}}}\begin{bmatrix} e^{-j\angle h_{1,1}} & \cdots & e^{-j\angle h_{N_{\mathrm{r}},1}} \\ \vdots & \ddots & \vdots \\ e^{-j\angle h_{1,N_{\mathrm{BS}}}} & \cdots & e^{-j\angle h_{N_{\mathrm{r}},N_{\mathrm{BS}}}} \end{bmatrix}\end{aligned} \tag{18}$$

*Proposition 1:* Assume that uniform linear arrays (ULAs) are equipped at the BS with $N_{\mathrm{BS}}\to\infty$ . Let AoD angles $\phi_{cl}\left(l\in[1,2,\ldots,L_u],c\in[1,2,\ldots,N_c]\right)$ uniformly distributed within $[0,2\pi]$, then the effective channel matrix $\hat{\mathbf{H}}$ is orthogonal almost surely.

*Proof:* Consider the SU system model depicted in Section 2, and the channel model in (3) and (5). According to (18), the diagonal elements of $\hat{\mathbf{H}}$ the can be given as

$$\hat{\mathbf{H}}\left[r,r\right]=1/\sqrt{N_{\mathrm{BS}}}\sum_{i=1}^{N_{\mathrm{BS}}}\left|h_{r,i}\right| \tag{19}$$

where $r\in\left[1,2,...,N_r\right]$. So $\hat{\mathbf{H}}\left[r,r\right]$ are real numbers because of phase offset after matrix multiplication. The non-diagonal elements of the effective channel matrix are

$$\hat{\mathbf{H}}\left[r,n\right]=1/\sqrt{N_{\mathrm{BS}}}\sum_{i=1}^{N_{\mathrm{BS}}}\left|h_{r,i}\right|\left(e^{j(\angle h_{r,i}-\angle h_{n,i})}\right)=\sum_{i=1}^{N_{\mathrm{BS}}}\mathrm{A}_i e^{j\phi_i} \tag{20}$$

where $\phi_i=\angle h_{r,i}-\angle h_{n,i},\mathrm{A}_i=1/\sqrt{N_{\mathrm{BS}}}\left|h_{r,i}\right|,r,n\in\left[1,2,...N_r\right]$. and $r\neq n$ . According to (20), $e^{j\phi_i}$ and $\mathrm{A}_i$ are linearly independent. $\mathrm{A}_i$ is the complex gain. The angle variable $\phi_i\in\left[0,2\pi\right]$ is uniformly distributed within $\left[0,2\pi\right]$, since the AoDs are selected from a uniformly distribution independently. So $\mathrm{E}\left[e^{j\phi_i}\right]=0$ if $N_{\mathrm{BS}}\to\infty$. We can denote that $\xi=\sum_{i=1}^{N_{\mathrm{BS}}}\mathrm{A}_i e^{j\phi_i}/N_{\mathrm{BS}}$ . Mathematically, the following formulas can be required according to the weak law of large number such as

$$\begin{aligned}&\lim_{N_{\mathrm{BS}}\to\infty}\left(\xi-E\left(\xi\right)\right)=0\\ &\Rightarrow\sum_{i=1}^{N_{\mathrm{BS}}}\mathrm{A}_i e^{j\phi_i}=\lim_{N_{\mathrm{BS}}\to\infty}E\left(\mathrm{A}_i e^{j\phi_i}\right)=N_{\mathrm{BS}}E\left(\mathrm{A}_i\right)\cdot E\left(e^{j\phi_i}\right)=0\\ &\Rightarrow\lim_{N_{\mathrm{BS}}\to\infty}\hat{\mathbf{H}}\left[r,n\right]=0\ .\end{aligned} \tag{21}$$

Consequently, the non-diagonal elements of the effective channel matrix are almost zeros and the diagonal elements are real numbers, so the matrix $\hat{\mathbf{H}}$ is orthogonal almost surely. As the number of antennas $N_{\mathrm{BS}}$ increases, the effective matrix $\hat{\mathbf{H}}$ becomes better conditioned.

Based on the above justification, EGT phased array for analogue precoding scheme is proposed in this paper. Diagonal elements can be efficiently maximized by designing $\mathbf{F_{RF}}$ and in the meantime, non-diagonal elements are minimized. Since $\mathbf{F_{RF}}$ is designed to maximize the diagonal elements of $\hat{\mathbf{H}}$, the non-diagonal elements are relatively small, especially whereas $N_{\mathrm{BS}}$ is large. Without loss of generality, $\varphi_c$ can be assumed to be random mean as $\varphi_c\in\left[0,2\pi\right]$ . So $\hat{\mathbf{H}}\left[k\right]=\mathbf{H}\left[k\right]\cdot\mathbf{F_{RF}}$ is almost orthogonal. As the analogue matrix has constant modulus elements, the phases of the elements are opposite to the phases of the channel matrix. Phases can be offset after matrix multiplication by employing the character of complex numbers.

The digital precoding is designed in the second step. In this digital step, water-filling approach [9] can be utilized. After analogue precoding, the effective channel matrix in each subcarrier is orthogonal almost surely. Therefore, the effective channel matrix $\hat{\mathbf{H}}\left[k\right]$ can be regarded as rows of parallel subchannels. Due to the way of water-filling power distribution, the Lagrangian multiplication can be used to allocate the subchannel power $P_s$, where $s\in\left[1,2,...N_s\right]$. In our proposed algorithm, $P_s$ is assigned for digital precoding matrix on corresponding diagonal elements. $\mathbf{F}_{\mathrm{BB}}\left[k\right]$, which can be got by water-filling is the diagonal matrix of $P_s$. After digital precoding on the effective channel, the total achievable sum rate of broadband mmWave MIMO systems can be maximized. And this water-filling algorithm has low computational complexity, since there is no requirement for SVD decomposition to obtain parallel subchannels.

When water-filling method is applied, the following power constraint $P_t$ should be satisfied as

$$P_t=\sum_{s=1}^{N_S}P_s=\sum_{s=1}^{N_S}\left(\mu-\sigma^2/\hat{\mathbf{H}}_{s,s}\right)^+ \tag{22}$$

where $\mu$ is the water-filling level. The digital precoder $\mathbf{F_{BB}}[k]$ is a diagonal matrix that is defined as

$$\mathbf{F_{BB}}[k]=\Lambda\cdot diag\left[P_1,P_2,\cdots,P_{N_S}\right] \tag{23}$$

where $\Lambda$ is a diagonal matrix which can be adjusted to satisfy the hybrid precoding power constraints as $\left\|\mathbf{F_{RF}}\mathbf{F_{BB}}[k]\right\|_F^2=N_S$ .

The proposed SU algorithm for broadband mmWave massive MIMO systems is as follows.

**Algorithm 1** The proposed SU algorithm

**Input:** $N_{\mathrm{BS}}$, $N_r$, $P_t$ and $\mathbf{H}[k]$.

**Output:** $\mathbf{F_{RF}}$ and $\mathbf{F_{BB}}$.

$$\mathbf{H}=\frac{1}{K}\sum_{k=1}^{K}\left(\mathbf{H}[k]\mathbf{H}[k]^{\mathrm{H}}\right)$$

**The analogue step:**

*for* $i$=1 to $N_{\mathrm{BS}}$ and $j$=1 to $N_{\mathrm{RF}}$ do

$$\mathbf{F_{RF}}[i,j]=1/\sqrt{N_{\mathrm{BS}}}\exp\left(-j\,\mathrm{angle}\left(h_{j,i}\right)\right),$$

end *for*

**The digital step:**

$$\mathrm{P}_s[k]=\mathrm{WaterFilling}\left(P_t,\mathrm{diag}\left(\hat{\mathbf{H}}[k]\right)\right),$$

where $\hat{\mathbf{H}}[k]=\mathbf{H}[k]\cdot\mathbf{F_{RF}}$, $s=1,2,...,N_S$.

$$\mathbf{F_{BB}}[k]=\mathrm{diag}\left(\mathrm{P}_s[k]\right),$$

**BS normalizes as**

$$\mathbf{F_{BB}}[k]=N_S\mathbf{F_{BB}}[k]/\left\|\mathbf{F_{RF}}\mathbf{F_{BB}}[k]\right\|_{\mathrm{F}},k=1,2,...,K.$$

Based on (6), $R[k]$ can be written to another type as $R[k]=\log_2\left|\mathbf{I}_{N_r}+\frac{1}{\sigma^2}\hat{\mathbf{H}}[k]\mathbf{F_{BB}}[k]\mathbf{F_{BB}^{H}}[k]\hat{\mathbf{H}}[k]^{\mathrm{H}}\right|$, where $\hat{\mathbf{H}}[k]=\mathbf{H}[k]\mathbf{F_{RF}}$. The digital precoder has a water-filling solution as $\mathbf{F_{BB}}[k]=\mathbf{\Gamma}[k]$, in which $\mathbf{\Gamma}[k]$ is the diagonal matrix of allocated powers to each symbol at subcarrier $k$.

Finally, we derive the upper-bound for the sum rate of SU in the objective of (7) based on Jensen's inequality as

$$\begin{aligned}\frac{1}{K}\sum_{k=1}^{K}R[k]&=\log_2\left|\mathbf{I}_{N_r}+\frac{1}{\sigma^2}\hat{\mathbf{H}}[k]\hat{\mathbf{H}}[k]^{\mathrm{H}}\mathbf{\Gamma}[k]\mathbf{\Gamma}[k]^{\mathrm{H}}\right|\\&\overset{(a)}{\le}\log_2\left|\mathbf{I}_{N_r}+\frac{1}{\sigma^2}\mathbf{F}_1\mathbf{\Gamma}[k]\mathbf{\Gamma}[k]^{\mathrm{H}}\right|\end{aligned}\tag{24}$$

where $\mathbf{F}_1=\frac{1}{K}\sum_{k=1}^{K}\hat{\mathbf{H}}[k]\hat{\mathbf{H}}[k]^{\mathrm{H}}$. (a) is according to Jensen's inequality, i.e., for a concave function $f(\cdot)$, if $\sum\alpha_i=1$, then $\sum\alpha_i f(\mathbf{X}_i)\le f\left(\sum\alpha_i\mathbf{X}_i\right)$.

## B. The Proposed MU Algorithm

In order to solve the problem (14), the following design scheme is considered in this paper: First, the analogue precoding matrix is developed whereas assuming that all users have equal priority. In other words, that means the effect of different priority weights and inter-user interference are neglecting in the first step design. The analogue precoding vector of each column in MU systems is better to be orthogonal because the channel state of all users can be improved. The analogue precoding matrix $\mathbf{F_{RF}}$ of MU is designed based on EGT method with the addition of a Gram-Schmidt orthogonalization process by maximizing (14). Second, the conventional digital scheme is utilized to remove the inter-user interference of the digital domain under the effective channel. The SVD decomposition of $\mathbf{HU}[k]$ in the $k^{\mathrm{th}}$ subcarrier is defined as $\mathbf{HU}[k]=\mathbf{U}[k]\Sigma[k]\mathbf{V}^{\mathrm{H}}[k]$. The digital precoder is given according to the ZF approach in [11] by $\mathbf{F_{BB}}[k]=\hat{\mathbf{H}}\mathbf{U}[k]^{\mathrm{H}}\left(\hat{\mathbf{H}}\mathbf{U}[k]\hat{\mathbf{H}}\mathbf{U}[k]^{\mathrm{H}}\right)^{-1}\left[\mathbf{V}[k]\right]_{1:N_s}\Lambda$, where $\hat{\mathbf{H}}\mathbf{U}[k]=\mathbf{HU}[k]\cdot\mathbf{F_{RF}}$ is effective channel matrix of MU, $\left[\mathbf{V}[k]\right]_{1:N_s}$ is the $N_s\times N_s$ matrix that gathers the $N_s$ dominant vectors of $\mathbf{V}[k]$, and $\Lambda$ is designed as a diagonal matrix, which can be adjusted to fulfill the hybrid precoding power constraint $\sum_{k=1}^{K}\left\|\mathbf{F_{RF}}\mathbf{F_{BB}^{\mathit{u}}}[k]\right\|_{\mathrm{F}}^2=KN_{\mathrm{S}},\ \forall u=1,2,\cdots U$.

*Remark 1:* The digital precoder in the unitary power constraint is given as $\mathbf{F_{BB}}[k]=\mathbf{A}_{\mathrm{BB}}[k]\mathbf{G}[k]$, where $\mathbf{A}_{\mathrm{BB}}[k]=\hat{\mathbf{H}}\mathbf{U}[k]^{\mathrm{H}}\left(\hat{\mathbf{H}}\mathbf{U}[k]\hat{\mathbf{H}}\mathbf{U}[k]^{\mathrm{H}}\right)^{-1}$ depends on the effective channel matrix, and $\mathbf{G}[k]\in N_s\times N_s$, which is called equivalent digital precoder is a semi-unitary matrix, as $\mathbf{G}^{\mathrm{H}}[k]\mathbf{G}[k]=\mathrm{I}_{N_s}$ can be verified. *Remark 1* indicates that it is necessary to know the analogue precoding, which solves (14), and the optimal equivalent semi-unitary matrix $\mathbf{G}[k]$, in order to obtain the optimal sum data rate with the unitary power constraint.

$\hat{\mathbf{H}}\mathbf{U}[k]$ is from the perspective of the analogue precoder in hybrid architectures. That brings the intuition that analogue precoding vectors to be orthogonal (or nearly orthogonal) is better. It means that $\hat{\mathbf{H}}\mathbf{U}[k]$ has a better coverage over the predominant subspace, which belongs to the real channel matrix. That intuition can be affirmed in *remark 1* as well. This observation is relevant to the solutions of the tight nearest frame and the nearest matrix problems [19],[20]. Based on that, in MU scenarios, $\mathbf{F_{RF}}$ in (16) is modified with the addition of a Gram-Schmidt orthogonalization process. That can be simply completed by multiplying the candidate precoding vectors with the projection matrix. The proposed MU precoding algorithm is summarized as follows.

(1).The analogue precoder $\mathbf{F_{RF}}$ is first derived from EGT.

(2).Gram-Schmidt orthogonalization process is employed in order to orthogonalize $\mathbf{F_{RF}}$. That can be completed by multiplying candidate vectors with the projection matrix $\mathbf{P}=\left(\mathrm{I}-\mathbf{F_{RF}}\left(\mathbf{F_{RF}^{H}}\mathbf{F_{RF}}\right)^{-1}\mathbf{F_{RF}^{H}}\right)$.

(3).The effective channel $\hat{\mathbf{H}}\mathbf{U}[k]$ of MU in the $k^{\mathrm{th}}$ subcarrier is denoted as $\hat{\mathbf{H}}\mathbf{U}[k]=\mathbf{HU}[k]\cdot\mathbf{F_{RF}}$.

(4).The digital precoder is defined as $\mathbf{F_{BB}}[k]=\hat{\mathbf{H}}\mathbf{U}[k]^{\mathrm{H}}\left(\hat{\mathbf{H}}\mathbf{U}[k]\hat{\mathbf{H}}\mathbf{U}[k]^{\mathrm{H}}\right)^{-1}\left[\mathbf{V}[k]\right]_{1:N_s}\Lambda$.

(5).$\left[\mathbf{V}[k]\right]_{1:N_s}$ is the $N_s\times N_s$ matrix that gathers the $N_s$ dominant vectors of $\mathbf{V}[k]$. $\Lambda$ is designed as a diagonal matrix to satisfy the hybrid precoding power constraint.

## C. Computational Complexity

We first compute the complexity of the proposed SU algorithm, which includes two parts as follows.

(1).The first part comes from the analogue precoder assignment. $N_{\mathrm{BS}} N_r$ matrix conjugate transpose is required.

(2).The second part is from the calculation of digital precoder. The complex multiplication of matrices is $\mathrm{O}\left(N_{\mathrm{BS}} N_r^2\right)$, and the power distribution requires $\mathrm{O}\left(N_r\right)$.

Consequently, the totally computational complexity of our proposed SU algorithm has a computational complexity of $\mathrm{O}\left(N_{\mathrm{BS}} N_r^2 + N_{\mathrm{BS}} N_r\right)$, which can significantly reduce the complexity compared to the full digital algorithm whose computational complexity is $\mathrm{O}\left(N_{\mathrm{BS}}^3\right)$. And the complexity of SU algorithm is much lower than the algorithm in [3] because of the search process of orthogonal basis.

Then, the complexity of the MU algorithm includes three parts because of the added orthogonalization as follows.

(1).The first part is the same as caculation of the SU algorithm, which contains $N_{\mathrm{BS}} U$ matrix conjugate transpose and $N_{\mathrm{BS}}$ phase value assignment.

(2).The second part is from the Gram-Schmidt orthogonalization that requires $N_{\mathrm{BS}} U^2$ multiplications and $U\left(U-1\right)/2$ divisions.

(3).The third part comes from ZF digital precoder. It requires $U^2$ multiplications and $\mathrm{O}(U^3)$ matrix inversion.

In summary, the totally computational complexity of the proposed MU algorithm is $\mathrm{O}\left(N_{\mathrm{BS}} U^2 + U^3\right)$. The iterative optimal algorithm in [6] has the computational complexity of $\mathrm{O}(N_{\mathrm{BS}}^3 U)$. Consequently, when $N_{\mathrm{BS}}$ is generally much lager than $U$, the complexity of MU algorithm is much lower than the full digital algorithm and the algorithm in [6].

## V. Simulation Results

In this section, the performance of the proposed algorithms for SU and MU is evaluated. We compare the SU algorithm with the full digital precoding, OMP-based precoding [3], and the MU algorithm with the full digital precoding, hybrid iterative optimal precoding [6].

The simulation parameters are depicted as follows. The broadband system model discribed in Section 2 and 3 are considered. ULA is implemented at BS. The antenna spacing of BS is half-wavelength. The number of subcarriers in both SU systems and MU systems is $K$=32. Further, unless otherwise mentioned, the channel of every subcarrier has 5 clusters and 10 propagation paths per cluster. The AoD and AOA of every propagation path is distributed in $\left[0, 2\pi\right]$ uniformly. The phase shift is random mean as $\varphi_c \in \left[0, 2\pi\right]$. The average power of every propagation path is randomly generated from a uniform random variable within [0,1]. And signal-to-noise ratio is defined as $SNR = \rho / \sigma^2$. For fairness, the whole same power constraint is imposed on all the above algorithms. All results are generated by 1000 trials Monte-Carlo simulation.

### A. Hybrid Precoding Design Analysis for SU

Here, we consider a SU system with $K$ = 32 subcarriers. Each subcarrier sends $N_s$ = 8 data streams to the MS with 8 antennas. And there are 8 RF chains in the BS.

Fig. 3 depicts the performance comparison of sum rate $R$, against the average $SNR$, with the $N_{\mathrm{BS}}$=128. It can be seen that $R$ in the proposed SU algorithm improves by increasing average $SNR$. And the sum rate achieved by SU algorithm can reach more than 90% of the full digital algorithm. The performance of the proposed algorithm only deteriorates about 2.5dB when fixing $R$ at 50bit/Hz/s. The proposed SU algorithm always outperforms the algorithms in [3].

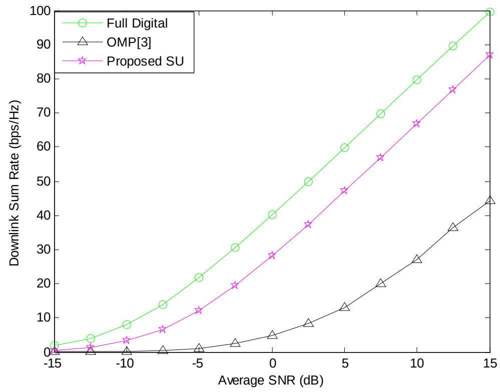


Figure 3. The Sum Rate vs. SNR with $N_{BS}$=128 for SU

Fig. 4 depicts the performance comparison of $R$ against the average $SNR$, with the $N_{\mathrm{BS}}$=256 anttenas of the BS. It can be seen that the sum rate can be enhanced whereas the number of $N_{\mathrm{BS}}$ increases. The sum rate of our proposed SU algorithm almost reaches 100bps/Hz at $SNR$ is 15dB. And the gap of performance between our proposed algorithm and those compared algorithms almost keeps consistent.

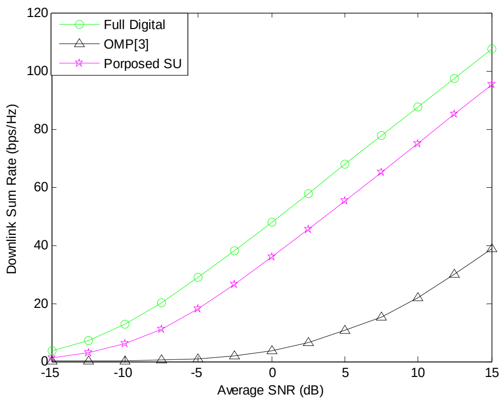


Figure 4. The Sum Rate vs. SNR with $N_{BS}$=256 for SU

### B. Hybrid Precoding Design Analysis for MU

Fig.5 depicts $R$ of our proposed MU algorithm against the average $SNR$, with 8 users at $N_{\mathrm{BS}}$=128. $U$ is the same as the number of $N_{\mathrm{RF}}$. We can see that the achievable sum rate of our proposed scheme and the algorithm in [6] converges to that of the fully digital algorithm when the number of antennas is large enough. However, The proposed MU algorithm always outperforms the algorithms in [6].

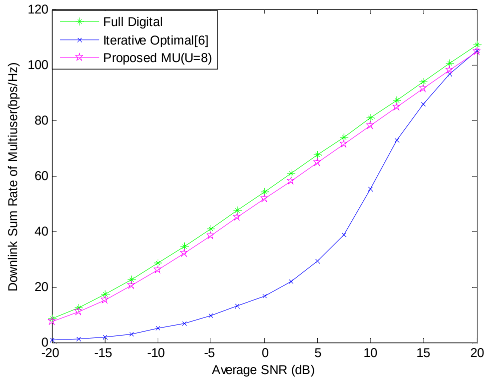


Figure 5. The Sum Rate vs. SNR with $U$=8 for MU

Fig.6 depicts the sum rate $R$ of our proposed MU algorithm against the average $SNR$, with 16 users at $N_{BS}$=128. Some observations can be obtained by analyzing Fig. 5 and Fig. 6. The first is the sum rate of the proposed MU improves by increasing users. The second is the sum rate of our algorithm improve by increasing RF chains owing to the diversity gains supplied by multiple RF chains.

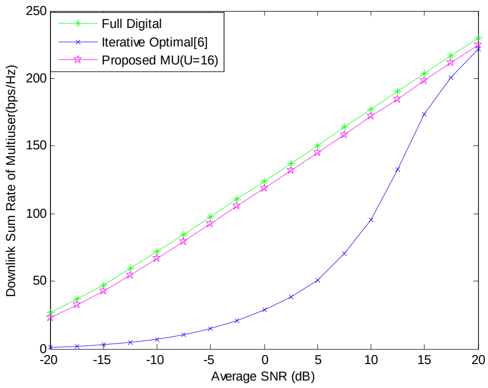


Figure 6. The Sum Rate vs. SNR with $U$=16 for MU

## VI. Conclusion

In this paper, we proposed hybrid SU and MU algorithms for broadband mmWave massive MIMO systems leveraging the immense deployed antennas with low-complexity. For SU systems, the EGT method is utilized to get the common analogue precoder. The digital precoder is obtained by directly water-filling method with no requirement for SVD decompositon to obtain parallel subchannels. For MU systems, the Gram-Schmidt orthogonalization is added to get the analogue precoder, and the digital precoder in MU system is designed according to ZF precoding in order to reduce interference. After the hybrid precoding, the total achievable sum rate can be maximized. Our algorithms have low computational complexity. The performances of the schemes under different configurations are shown from various simulation results. Our proposed algorithms approach to the performance of the full digital precoding algorithm and get better performance compared to other hybrid algorithms.

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