A Multi-objective PMU Placement Method Considering Observability and Measurement Redundancy using ABC Algorithm

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***Abstract*—This paper presents a Multi- objective Optimal Placement of Phasor Measurement Units (MOPP) method in large electric transmission systems. It is proposed for minimizing the number of Phasor Measurement Units (PMUs) for complete system observability and maximizing the measurement redundancy of the system, simultaneously. The measurement redundancy means that number of times a bus is able to monitor more than once by PMUs set. A higher level of measurement redundancy can maximize the total system observability and it is desirable for a reliable power system state estimation. Therefore, simultaneous optimization of the two conflicting objectives are performed using a binary coded Artificial Bee Colony (ABC) algorithm. The complete observability of the power system is first prepared and then, single line loss contingency condition is considered to the main model. The efficiency of the proposed method is validated on IEEE 14, 30, 57 and 118 bus test systems. The valuable approach of ABC algorithm is demonstrated in finding the optimal number of PMUs and their locations by comparing the performance with earlier works.**

***Index Terms*—artificial bee colony algorithm, complete observability, measurement redundancy, optimal placement, phasor measurement unit.**

I. INTRODUCTION

A. Analysis of present trend of the power system

In present days, power system deregulation, uneven expansion of power generation and demand and other things have burdened the existing power system and decreased their stability level [1]. The unexpected rise of non-linear loads such as heaters, air conditioners and motors leads to the voltage instability problem. As a result of this present trend and economical growth, power systems worldwide have become increasingly troubled with the voltage stability and collapse problems [2]. The blackouts of 14th August 2003 in New York and June 30 and 31st 2012, in India, are some of the well known examples, connected with voltage collapse problems. In order to avoid the occurrence of such blackouts, it is very important to consider the maximum loadability limit of the power systems [3]. In such conditions, to ensure stable and proper operations, an exact measurement and observation of the power system states are needed. It is achieved by utilizing the Supervisory Control And Data Acquisition (SCADA) system, in which data collected by Remote Terminal Units (RTUs) is not usually synchronized. In addition, SCADA could not supply any information about the dynamic state of the power system [4]. To overcome these limitations of SCADA, the Wide-Area Monitoring, Protection And Control (WAMPAC) system has been used, in which PMU is considered as a basic unit.

B. Analysis of phasor measurement units

A PMU is a power system monitoring device which was introduced in 1980s that presents fast and smart communications. It plays a main role in a smart grid. The key characteristics of PMUs are measurements of voltage and current phasor that are time-stamped through a Global Positioning System (GPS), which aids to synchronize the data, taken from different locations in widely dispersed power system. PMUs have better accuracy than SCADA system because of sampling the input waveforms with a higher rate [4]. The synchro phasor technology brings the opportunities to enhance the real time monitoring, fault location analysis, protection and control of the power system [5]. Then, sampled data is transmitted through optical fiber to the phasor data concentrator (PDC), which is installed at the centralized location, for taking proper actions to remain a stable system. The PDC aligns the time of the data and presents it to the historian. It records the data for retrieval and post-dispatch analysis of any grid event. After getting the experience of main blackouts around the world, mainly on 14th August 2003, U.S.A blackout creates a new force for implementation of Wide-Area Measurement Systems (WAMS) using PMUs. Presently, determining the optimal number of PMUs and their locations in transmission systems has become main activity.

C. Analysis of optimal PMUs placement methods

The Optimal PMUs Placement (OPP) methods can be categorized as Conventional and Heuristic optimization method [6]. The conventional method such as Integer Linear Programming (ILP) method is applied to the OPP problem. In [7- 8], the authors used integer programming to determine the minimum number of PMUs. But, this technique may experience from the difficulty of being trapped into local minima. One more shortcoming is that, starting from a first guess, it may direct to only one solution, while more than one solution may be present. Therefore, multiple objective problems cannot be handled by integer programming [9]. The stochastic search algorithms such as Genetic Algorithm (GA) [10], Immunity Genetic Algorithm (IGA) [11], Binary Particle Swarm Optimization (BPSO) [12] and Improved

Digital Object Identifier 10.4316/AECE.2014.02020

Tabu Search (ITS) [13] are presented to obtain the reliable OPP solutions. Also other solution techniques such as fuzzy-based procedure for multi- objective problem [14], Evolutionary optimization algorithm [15], Cultural algorithms [16], Cat swarm optimization algorithm [17], PSO-DP [18], and Adaptive Neuro- Fuzzy Inference System and Genetic Algorithm [19] are applied for different power system optimization problems and is found to be successful. These techniques are found to be good for determining the global optimal solutions of PMUs placement and can be considered successful to a certain extent. As a novel swarm based optimization techniques are observed as a promising one, the best location with less computation time is a tough task within the research field. An innovative optimization technique recognized as Artificial Bee Colony (ABC) algorithm, suggested by Karaboga in 2005, was effectively implemented to various applications such as Engineering Design Optimization Problems [20], Constrained Optimization Problems [21], Numerical Optimization [22] and Optimal Distributed Generation Allocation [23]. The performance and literature reviews of ABC algorithm and applications are available in the literatures [24-26].

In our earlier work, ABC algorithm was tested and validated on power system optimization problems such as Dynamic Economic Dispatch Problem [27], Unit Commitment Problem [28-29]. In this situation, an effort is taken to solve the multi-objective OPP problem, using a binary coded ABC algorithm in order to enhance both the observability and the measurement redundancy of the system.

II. Proposed Work

The aim of this paper is to prove the efficiency of ABC algorithm for solving the Multi- objective OPP (MOPP) problem. The major work of this paper is that minimizing the number of PMUs for complete system observability and maximizing the measurement redundancy, simultaneously.

The remaining part of the paper is ordered as follows. Section III, presents the proposed MOPP problem formulations with the constraints. Section IV, provides a summary of ABC algorithm. Section V, discusses the implementation of the ABC algorithm to solve the MOPP problem. Section VI, presents the proposed method results of IEEE test systems for different conditions of the power system. Finally, the conclusion is discussed in Section VII.

III. OPP Problem Formulation

In this proposed method, the optimal solution is achieved, by considering the zero injection bus constraint to minimize the number of PMUs for complete observability and bus measurement redundancy constraint to maximize the system observability. The placement of PMUs on all buses permits the direct measurement of the power system state. But, PMU placement on each bus is difficult to accomplish either due to the high cost of PMUs or unavailability of communication facilities in some places. The vital rule of PMU placement is that, when a PMU is placed on a bus, it can measure the voltage phasor on that bus, as well as on the buses on the other end of all the connected lines, using the measured current phasor and the known transmission line impedance [9]. Obviously, the installation of PMU makes that bus directly observable and its neighboring buses indirectly observable, by applying Ohm's law [8, 30-31]. This implies that a system can be made observable with a minimum number of PMUs than the number of buses by using the zero injection bus constraint. A higher level of measurement redundancy can maximize the system observability and it is necessary for a reliable power system state estimation. To estimate the capability of the proposed MOPP problem formulation, optimal PMU placement for the 14, 30, 57 and 118 bus IEEE standard systems is achieved.

A. Minimizing the number of PMUs

The objective function stands for PMUs installation in a system. The observability of a bus depends on the installation of PMU on that bus or one of its incident buses. If a bus is said to be topologically observable, when there is at least one measurement on that bus either directly or indirectly. Minimizing the number of PMUs for achieving a complete observability of the power system can be expressed as follows:

$$n_{pmu} = Min \sum_{i=1}^{N} p_i \tag{1}$$

Subject to the following constraints,

$$J * P \geq I \tag{2}$$

$$c_{i,j} = \begin{cases} 1, \text{ if i=j;} \\ 1, \text{ if i and j are adjacent buses;} \\ 0, \text{ otherwise;} \end{cases} \tag{3}$$

$$P = [p_1 \; p_2 \; \dots \; p_N]^T \tag{4}$$

$$p_i = \begin{cases} 1, \text{ if a PMU is installed at bus i;} \\ 0, \text{ otherwise;} \end{cases} \tag{5}$$

where n_{pmu} is the minimum number of PMUs required, N is the total number of buses in a system, J is the connectivity matrix of the system with a size of $N*N$, which consists of binary element $c_{i,j}$, P is the optimal PMUs placement vector, I is the observability vector with a size of $N*1$, P should be equal to one or greater than one for complete system observability, p_i defines the possibility of a PMU on a bus which is the binary variable determined using the proposed ABC algorithm.

B. Without considering zero injection bus constraint

Zero injection bus is a bus, which doesn't have any generator or load. If zero injection bus constraint is not considered in the Optimal PMU Placement problem formulation, then each bus should be observed at least a time, either directly or indirectly through a PMU, for complete observability of the system. Then, observability vector (U) in Eq. (2) becomes unit vector I, with a size of $N*1$.

$$I = [11 \dots 1]^T \tag{6}$$

C. With considering zero injection bus constraint

In OPP problem, required number of PMUs can be further minimized by considering the zero injection bus constraint.

The detailed explanation can be found in [30]. When the zero injection bus constraint is incorporated into the OPP problem, Eq. (2) can be rewritten as Eq. (7). It is considered with the base case OPP problem formulation and is given by Eqs. (7-10).

$$J * P \geq K \tag{7}$$

$$R_j = 1 \; \forall j \notin B_1 \cup B_2 ... B_{NZ} \tag{8}$$

$$\sum_{k \in B_z} R_k \geq \left| A_z \right| \forall z \in S_z \tag{9}$$

$$B_z = A_z \cup \{z\} \tag{10}$$

where z is a zero injection bus, A_z is the set of buses adjacent to bus z, B_z is the set of adjacent buses to z and itself, NZ is the total number of zero injection buses in a system, S_z is the set of zero injection buses in a system, K is the observability vector with a size of $N*1$, not a unit vector under zero injection bus consideration.

When buses, which are connected to the zero-injection bus, all are observable except a bus, the unobservable bus will be observed by applying KCL at zero-injection bus (i.e., zero injection bus effect) [8, 32-34]. Therefore, it is not necessary to monitor all the zero injection buses or one of them adjacent buses. Hence, it minimizes the number of PMUs required for complete observability of the system.

D. Maximizing the system observability

When a PMU is located in a bus, it can present real time-synchronized measurement of voltage phasor on that bus and also current phasor, at the connected branches of the bus [34]. It is assumed that the PMUs have an adequate number of channels to measure the current phasors of all transmission lines, incident to the bus, at which they are placed.

The OPP problem does not have a single solution using this basic objective function. The optimization algorithms may result in different sets of solutions with the same minimum number of PMUs. Hence, Total System Observability Redundancy Index (TSORI) and measurement redundancy may be considered as a constraint in solving the OPP problem. A bus is said to be observable, when there is at least one measurement on that bus either directly or indirectly, through a PMU.

For solving the OPP problem, TSORI is used to calculate the total system observability by adding the Bus Observability Index (BOI) of all the buses in a system. The BOI is maximized randomly while keeping the same minimum number of PMUs which is obtained in base case OPP problem as in [30].

$$TSORI = \sum_{i=1}^{N} BOI_i \tag{11}$$

$$1 \leq BOI_i \leq \left(\tau_i + 1\right) \tag{12}$$

$$Max\left(BOI_i\right) \text{ or } Min\ \left(N - BOI_i\right) \tag{13}$$

$$\sum_{i=1}^{N} p_i = n_{pmu} \tag{14}$$

where $TSORI$ represents the total system observability redundancy index, which indicates the level of system observability, BOI_i represents the number of PMUs which can monitor the bus i, τ_i represents the maximum connectivity of the bus i. BOI_i value always lies between one and the maximum connectivity of the bus i (τ_i), It is considered for maximizing the total system observability in all cases.

E. Formulation of Multi-objective function

Most of the power system optimization problem involves the simultaneous optimization of several objective functions. These objective functions are non-commensurable, competing and conflicting in nature. The practical multi-objective optimization problem is given as follows:

$$F(x) = Min\ [F_1(x), F_2(x), ...F_n(x)] \tag{15}$$

Subject to the constraints,

$$h_e\left(x\right) \leq 0,\ e = 1,2,...\ \mathrm{T}_n \tag{16}$$

$$y_m\left(x\right) = 0,\ m = 1,2,...\ \mathrm{H}_n \tag{17}$$

where $F_1(x)$, $F_2(x)$ ... $F_n(x)$ are different objective functions, T_n is the number of equality constraint, H_n is the number of inequality constraints.

Multi-objective optimization method has such conflicting objective functions that give rise to a set of optimal solutions instead of single solution, since no solution can be considered better than any other, with respect to all objective functions. In this paper, two conflicting objective functions are considered and formulated as a constrained multi-objective function, given as follows:

$$Min\ \left[n_{pmu},\ N - BOI_i\right] \tag{18}$$

Subject to the constraints of Eqs. (2-14).

IV. Summary of ABC Algorithm

In 2005, Karaboga suggested an Artificial Bee Colony (ABC) algorithm [22], to optimize the numerical problems. ABC algorithm was developed based on inspecting the behaviors of real bees on finding nectar and sharing the information of food resources to the bees through Waggle Dance in the hive and is shown in Fig.1.

It has four necessary components such as Employed bees, Unemployed bees, Dancing area, Food resources. The employed bees select resources of food based on their practice. Once it returns to hive, the employed bee transfers the information of food resources to the unemployed bee through waggle dance. Based on the information gained from the employed bee, unemployed bee will search the food resource. All food resource selected represents a feasible solution to the problem in this concern.

Once the employed bees return to the hive, three types of information are transformed to the unemployed bee regarding the food resources through waggle dance.

First one is direction of food resources. Here, if the employed bee danced in the direction of 65 degree to the sun, then the unemployed bee starts to move towards 65 degree to the sun which is shown in Fig. 2.a.

Second one is the distance of food resources. If the employed bee moved with the vibration over 1 sec, then the unemployed has to travel one kilometer towards the above direction as shown in Fig. 2.b.

Finally, the information about the quality of food resources is transferred between them by releasing the alkanes from their abdomen into the air. If more alkanes are released then the quality of food resources is higher as shown in Fig. 2.c. If there is no improvement in the food source (fitness) then the scout bees fly and select the food sources randomly without using experience.

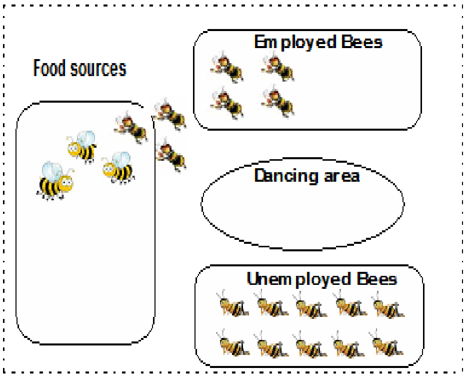

Figure 1. Main components of ABC algorithm

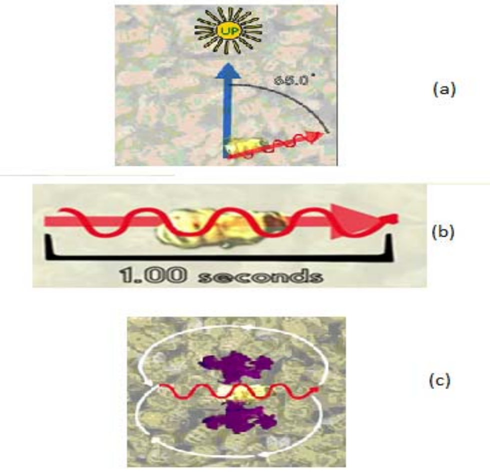

Figure 2. Behavior of artificial bee colonies

A. Pseudo- code for ABC Algorithm

Based on this model, Karaboga suggested the common Pseudo- code for ABC algorithm which is specified below.

Step 1: Give the system information. Set the ABC control parameters.

Step 2: Initialize the colony with random solutions.

Step 3: Estimate the Fitness function. Cycle begins.

Employed Bee phase:

Step 4: Employed bees adjust their locations using the greedy selection principle (GSP).

$$D_{pq} = x_{pq} + \beta_{pq}(x_{pq} - x_{fq}) \tag{19}$$

where $f \in \{1, 2 \ldots m\}$ and $q \in \{1, 2 \ldots L\}$ are arbitrarily selected indices, but f is calculated arbitrarily and is dissimilar from p, m is the total number of bees' locations and L is the number of optimized parameters. Here, x_{pq} is the location of bee, β_{pq} is an arbitrary number within (-1, 1). It manages the making of adjacent food resources. If the nectar sum of the new resource is better than prior one, the onlookers memorize the new location; or else, it maintains the older location.

Step 5: Employ the onlooker bees for selected sites.

$$Cho_p = \frac{FIT_p}{\sum_{k=1}^{m} FIT_k} \tag{20}$$

FIT_p is the fitness value of the solution p, which is proportional to the nectar sum of the food resource in the bee location p, and m is the total number of bees' locations.

Unemployed Bee phase:

Step 5: Onlookers adjust their locations with respect to bee position p and determine the fitness value.

Step 6: Adapt the position by the onlookers with respect to bee position p and evaluate the objective functions.

$$D_{pq} = x_{pq} + \beta_{pq}\left(x_{pq} - x_{fq}\right) \tag{21}$$

Step 7: Discard the resources used by the bees. Scout bee finds a new solution using an Eq. (22).

$$x_{pq} = x_{q\min} + \beta_{pq}\left(x_{q\max} - x_{q\min}\right) \tag{22}$$

Here, x_{qmin} and x_{qmax} are the minimum and maximum restrictions of the parameter should be optimized. The limit value generally varies from 1 to 10.

Step 8: Remember the best solution (quality of food) attained until now.

Step 9: Stop the procedure if the execution criteria is met. if not, go to step 4.

V. Implementation of ABC Algorithm

Here, ABC algorithm is implemented to find out the optimal number of PMUs and their locations for minimizing the number of PMUs and maximizing measurement redundancy of the system simultaneously as shown in Fig. 3.

A. Conversion of real to binary coded ABC algorithm

In order to convert from real coded algorithm to discrete space, the probability value is calculated from the position of each bee to determine whether x_{pq} will be in an ON or OFF state (0 or 1). They squashed D_{pq} using the following Eqs. (23- 24) as in [28].

$$\Pr(D_{pq}) = \frac{1}{1 + \exp(-D_{pq})} \tag{23}$$

$$x_{pq} = \begin{cases} 1, & \text{if } \beta_{pq} < \Pr(D_{pq}) \\ 0, & \text{otherwise} \end{cases} \tag{24}$$

VI. Case Studies And Discussion

The proposed optimal PMU placement method is performed using ABC algorithm. Programs are written using MATLAB 7.01 by Pentium IV processor 3.2 GHz speed and 1GB RAM. The effectiveness of the ABC algorithm for the proposed method have been verified on standard IEEE 14, 30, 57 and 118 bus systems. Table I shows the different cases of OPP problems considered here. All the three cases are solved using proposed ABC algorithm. To validate a proposed ABC algorithm, Case 1 is solved using the existing standard GA and PSO algorithm and the results are compared with proposed ABC. Table II shows the specifications of IEEE test systems [3, 32-33, 35]. In MOPP problem, maximizing measurement redundancy is considered randomly on the buses. The maximizing measurement redundancy value has the advantage that a major portion of the system will remain observable, in case one of the PMUs fails.

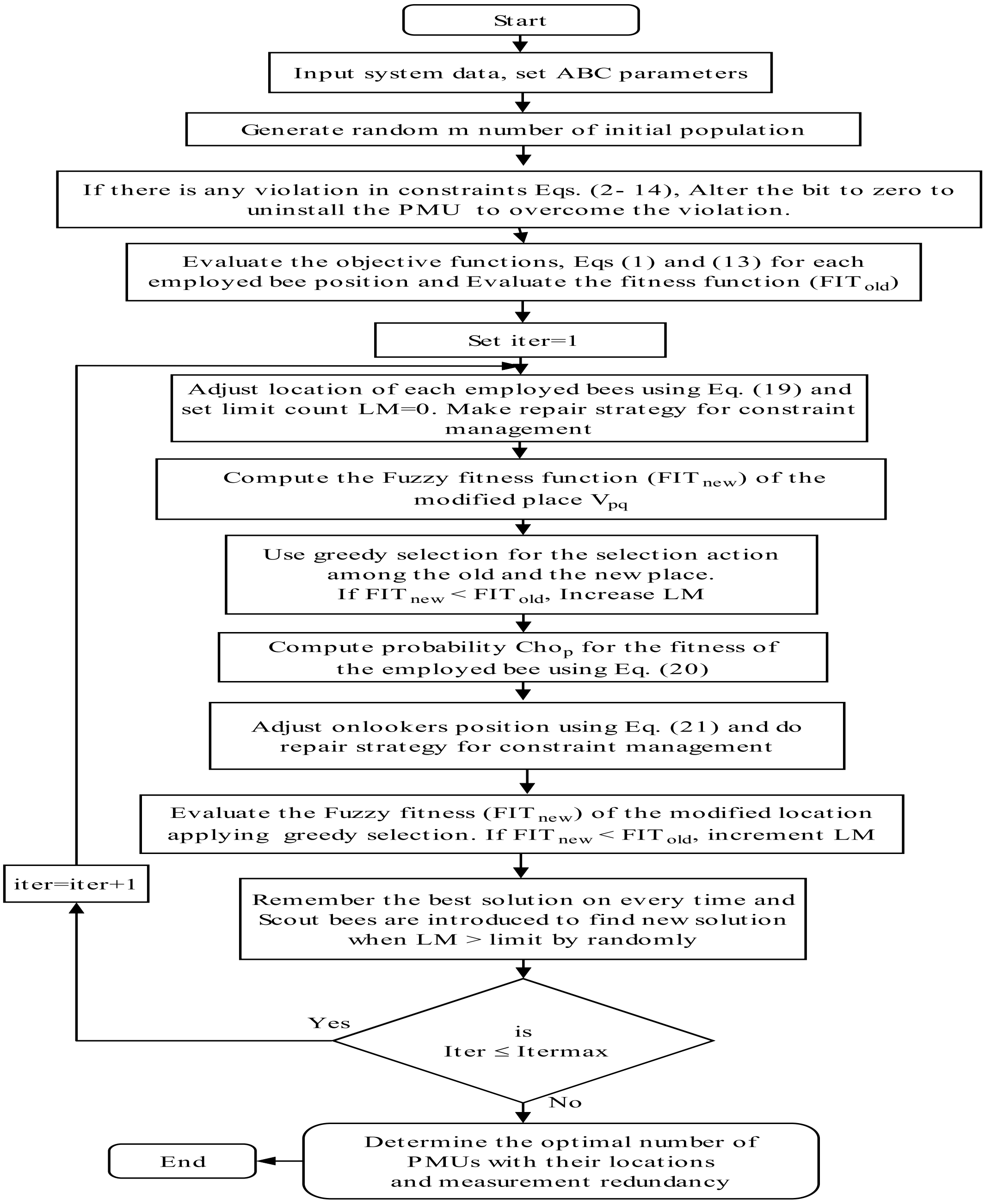

Figure 3. Flow chart for proposed FBABC method for solving MOPP problem

TABLE I. DIFFERENT CASES OF OPP PROBLEMS

Case	**Description**
1	To validate the ABC without considering zero injection bus constraint for normal operating condition
2	To validate the ABC with considering zero injection bus constraint for normal operating condition
3	To validate the ABC with considering zero injection bus constraint for single line loss

TABLE II. SPECIFICATIONS OF IEEE SYSTEMS [3, 32- 33, 35]

IEEE system	**No. of zero injection buses, (Z)**	**Set of zero injection buses, (S_z)**	**Max. no. of branches Connected to a bus, (τ)**
14 bus	1	7	5
30 bus	6	6, 9, 22, 25, 27, 28	7
57 bus	15	4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40 45, 46, 48	6
118 bus	10	5, 9, 30, 37, 38, 63, 64, 68, 71, 81	12

A. Parameters settings of ABC algorithm

In order to keep away from the confusing results due to foraging behavior of the real bees, several test runs are carried out to set the colony size and the limit value. Thirty trials are run for each set, with each run starting with a different random colony size. The colony size is varied from 20 to 200 and the limit value is varied from 1 to 10, suitably in equal intervals. The maximum iteration is set as 500.

Table III shows the selected value of ABC control parameters for solving the PMUs placement problem. These values are selected after 30 trials of this algorithm and offer the best performance in terms of finding the optimal locations of PMUs.

TABLE III. CONTROL PARAMETERS OF ABC ALGORITHM

IEEE system	Limit value	ABC colony size
14 bus	3	40
30 bus	6	100
57 bus	2	200
118 bus	2	200

B. Case 1

In this case, MOPP problem is solved using ABC algorithm for normal state of the power system, not including the zero injection bus constraint. It should be noted that the obtained results are equal to the optimal number reported in the available literatures [8-9, 30, 36-37]. It justifies that the proposed ABC algorithm can be able to produce a better quality solution for solving the MOPP problem.

Table IV shows the different solutions for IEEE 14 bus system. From Table IV, it is observed that five different solutions are obtained from thirty trials. It is also clear that, the first PMUs set gives maximum system observability than other PMUs sets.

The measurement redundancy is defined as the number of times a bus is observed more than a time by PMUs set**.** The maximizing bus measurement redundancy constraint has the benefit that a larger portion of the power system will remain observable, in case, one of the PMUs fails. From Table V, it is clear that the locations of PMUs, i.e., 2, 6, 7 and 9 are more desirable to guarantee the complete system observability and also enhance it.

In order to compare the robustness and calculation efficiency of the ABC algorithm, the MOPP problem is solved for IEEE 14 bus system using typical GA and PSO algorithm. Therefore, the initial random generated population is kept same for GA, PSO and ABC algorithm. The comparison of the computation efficiency of the three algorithms for IEEE 14 bus system is given in Table VI. It is concluded that the ABC algorithm can offer best solutions of MOPP with less computation time than GA. It should be noted that the PMUs placement problem is an off-line procedure. Therefore, the execution time is not much important in the MOPP problem.

Fig. 4 shows the distribution of optimal number of PMUs and their locations obtained from 30 trial runs. It should be noted that the frequency of achieving the minimum number of PMUs with maximum measurement redundancy value is higher, in case of ABC algorithm, when compared with GA and PSO techniques. It shows the superiority of ABC algorithm over other solution techniques.

Fig. 5 shows the convergence characteristic of GA, PSO and ABC for IEEE 14 bus system. It is clear that the characteristic of ABC is gradually reaching the minimum value after a few iterations and generates better quality solutions. It is proved that ABC is computationally effective in giving quality solutions obtained in minimum computation time at par with GA and PSO.

Table VII shows the different solutions obtained from 30 trials run for IEEE 30 bus system. It is shown that numbers of buses are observed more than a time is higher in the first PMUs set than any other sets in Table VII. So, it is the best solution according to optimal PMU objective. The first PMUs set is similar to optimal solution in [4], fifth PMUs set is similar to [9], and the second PMUs set is similar to [38], but [38] has only 12 buses which are observed more than a time. So, it is not a good solution. The best MOPP set is the first PMUs set in Table VII and have 14 buses which observed more than a time even though TSORI value is lower than TSORI value of second to fourth PMUs set.

Table VIII shows the different solutions for IEEE 57 bus system. First two PMUs sets show the best results according to the optimal PMU placement method. These solutions have same minimum number of PMUs and maximum measurement redundancy. The optimal PMUs placement set shown in second PMUs set of Table VIII is similar to the research work carried out in [4].

TABLE IV. DIFFERENT SOLUTIONS FOR IEEE 14 BUS- CASE 1

Location of PMUs, (*P*)	No. of PMUs can observe the bus, (*BOI*)														*TSORI*	No. of buses observed more than a time
	1	2	3	4	5	6	7	8	9	10	11	12	13	14		
2, 6, 7, 9	1	1	1	3	2	1	2	1	2	1	1	1	1	1	19	4
2, 7, 10, 13	1	1	1	2	1	1	1	1	2	1	1	1	1	1	16	2
2, 6, 8, 9	1	1	1	2	1	1	2	1	1	1	1	1	1	1	16	3
2, 7, 11, 13	1	1	1	2	1	2	1	1	1	1	1	1	1	1	16	2
2, 8, 10, 13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	14	0

TABLE V. MEASUREMENT REDUNDANCY ANALYSIS- CASE 1

Locations of PMUs, (*P*)	Measurement redundancy														No. buses observed more than a time
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
2, 6, 7, 9	0	0	0	2	1	0	1	0	1	0	0	0	0	0	4
2, 7, 10, 13	0	0	0	1	0	0	0	0	1	0	0	0	0	0	2
2, 6, 8, 9	0	0	0	1	1	0	1	0	0	0	0	0	0	0	3
2, 7, 11, 13	0	0	0	1	0	1	0	0	0	0	0	0	0	0	2
2, 8, 10, 13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE VI. COMPARISON OF ALGORITHMS- IEEE 14 BUS SYSTEM- CASE 1

Algorithm	Mean time (in seconds)	Frequency of achieving the minimum number of PMUs with maximum measurement redundancy
GA	120	11
PSO	40	17
ABC	40	21

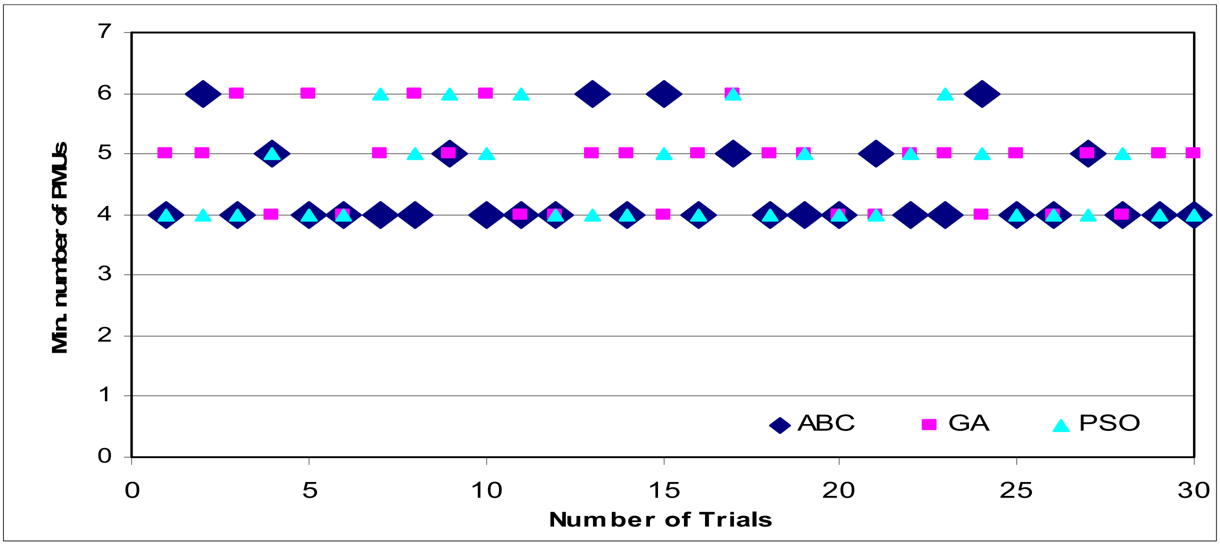

Figure 4. Distribution of PMUs in optimal locations - IEEE14 bus- Case 1

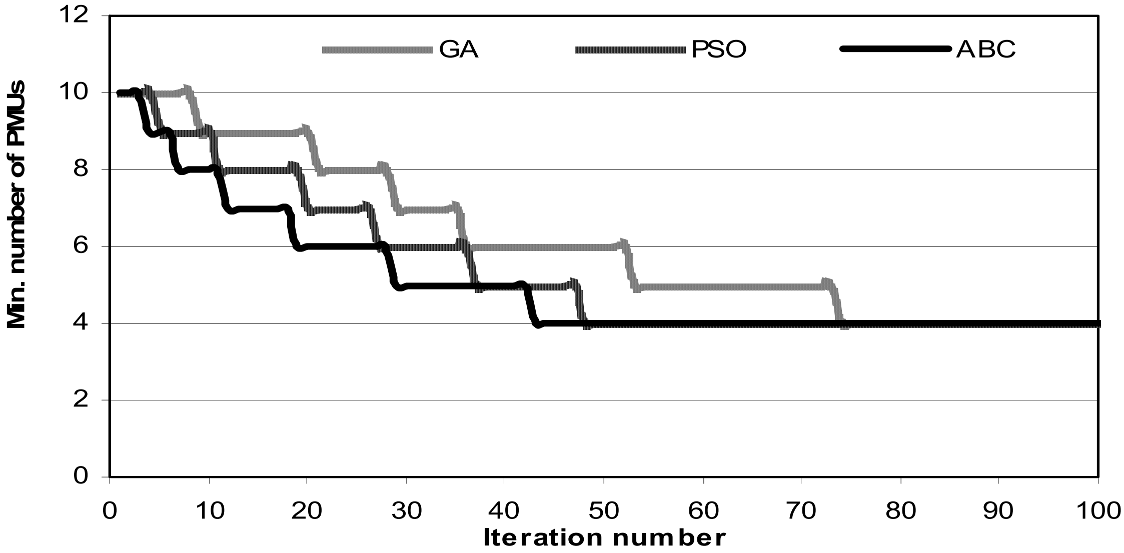

Figure 5. MOPP problem convergence graph - IEEE 14 bus - Case 1

TABLE VII. DIFFERENT SOLUTIONS FOR IEEE 30 BUS- CASE 1

Solution	**Locations of PMUs, (*P*)**	***TSORI***	**No. buses observed more than a time**
1	1, 2, 6, 9, 10, 12, 15, 19, 25, 27	50	14
2	2, 4, 6, 9, 10, 12, 15, 18, 25, 27	52	12
3	2, 4, 6, 9, 10, 12, 15, 19, 25, 27	52	13
4	2, 4, 6, 9, 10, 12, 15, 20, 25, 27	52	12
5	2, 4, 6, 9, 10, 12, 18, 24, 25, 27	51	12
6	2, 4, 6, 9, 10, 12, 19, 24, 25, 27	51	12
7	2, 4, 6, 10, 11, 12, 15, 20, 25, 27	50	12
8	2, 4, 6, 10, 11, 12, 19, 24, 25, 27	49	12
9	2, 3, 6, 9, 10, 12, 15, 20, 25, 27	48	11
10	1, 5, 6, 9, 10, 12, 15, 20, 25, 27	48	13
11	1, 2, 6, 10, 11, 12, 15, 18, 25, 27	48	13
12	1, 2, 6, 10, 11, 12, 15, 20, 25, 27	48	13
13	1, 5, 6, 9, 10, 12, 18, 24, 25, 27	47	12
14	1, 2, 6, 10, 11, 12, 19, 24, 25, 27	47	12
15	2, 4, 6, 9, 10, 12, 19, 23, 26, 29	46	8
16	1, 5, 6, 9, 10, 12, 18, 23, 26, 27	44	9
17	3, 6, 7, 10, 11, 12, 15, 19, 25, 29	44	11

TABLE VIII. DIFFERENT SOLUTIONS FOR IEEE 57 BUS- CASE 1

Solution	Locations of PMUs, (*P*)	*TSORI*	No. buses observed more than a time
1	1, 4, 6, 9, 15, 20, 24, 28, 31, 32, 36, 38, 41, 46, 51, 53, 57	72	15
2	1, 4, 6, 9, 15, 20, 24, 25, 28, 32, 36, 38, 41, 47, 50, 53, 57	72	15
3	1, 4, 7, 9, 15, 20, 24, 25, 27, 32, 36, 38, 39, 41, 46, 50, 53	71	13
4	1, 6, 9, 15, 19, 22, 25, 26, 29, 32, 36, 38, 41, 47, 50, 53, 57	71	14
5	1, 6, 13, 15, 19, 22, 25, 27, 32, 36, 38, 41, 47, 51, 52, 55, 57	70	12

TABLE IX. DIFFERENT SOLUTIONS FOR IEEE 118 BUS- CASE 1

Solution	Locations of PMUs, (*P*)	*TSORI*	No. buses observed more than a time
1	3, 5, 9, 12, 15, 17, 21, 23, 28, 30, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68 71, 75, 77, 80, 85, 86, 91, 94, 101, 105, 110, 114	164	39
2	3, 5, 9, 12, 15, 17, 20, 23, 28, 30, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68 71, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114	164	38
3	3, 5, 9, 12, 15, 17, 20, 23, 25, 28, 34, 37, 42, 45, 49, 52, 56, 62, 64 68, 71, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110, 115	162	37
4	2, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 42, 45, 49, 53, 56, 62, 64, 68, 70 71, 75, 77, 80, 85, 86, 91, 94, 101, 105, 110, 114	162	36

TABLE X. OPTIMAL SOLUTIONS- CASE 1

IEEE system	Min. no. PMUs	Locations of PMUs, (*P*)	n_{pmu}/N, %	*TSORI*
14 bus	4	2, 6, 7, 9	28 %	19
30 bus	10	1, 2, 6, 9, 10, 12, 13, 19, 25, 27	33 %	50
57 bus	17	1, 4, 6, 9, 15, 20, 24, 25, 27, 32, 36, 38, 39, 41, 46, 50, 53	29 %	72
118 bus	32	3, 5, 9, 12, 15, 17, 21, 23, 28, 30, 34, 37, 40, 45, 49, 52, 56, 62 64, 68, 71, 75, 77, 80, 85, 86, 91, 94, 101, 105, 110, 114	27 %	164

TABLE XI. COMPARISON OF PROPOSED RESULTS- CASE 1

Method	Min. no. of PMUs, (n_{pmu})			
	14 bus	30 bus	57 bus	118 bus
Proposed	4	10	17	32
ILP, [8]	4	10	17	32
Binary search, [9]	4	10	-	-
ILP, [30]	4	-	17	32
BILP, [37]	4	-	-	32
ILP, [38]	4	-	17	32

The MOPP solution of third PMUs set was the best result in [38], but actually it is not a good solution, because it consists only 13 buses, which are observed more than a time by this MOPP set. The proposed method provides the best results for MOPP and satisfies all the conditions of MOPP objective. IEEE 57 bus test systems consists of two best solutions set, as in first two PMUs sets of Table VIII, and have 15 buses which observed more than a time.

Table IX shows the different PMUs placement sets obtained from 30 trials run for IEEE 118 bus system. First PMUs set of Table IX is similar to optimal solution given in [4] and has maximum measurement redundancy. It is clear that, the MOPP consists of 39 buses which are observed more than a time, since it is the best solution.

The optimal locations are determined in such a way that which PMUs set offers the minimum number of PMUs and maximum measurement redundancy for complete system observability, for normal power system operating conditions. Here, maximizing bus measurement redundancy is considered, randomly on the buses. Therefore, total system observability is improved. The results presented in Table X guarantee the complete observability of the system with minimum number of PMUs and maximum measurement redundancy.

The proposed method demonstrates the minimum number of PMUs for complete power system observability, when the proposed result is compared with earlier works, as shown in Table XI. It should be noted that the obtained results are equal to the optimal number reported in the available literatures in [8-9, 30, 36-37] but proposed method has higher measurement redundancy.

C. Case 2

Here, MOPP problem is solved using ABC algorithm, considering the zero injection bus constraint. The optimal solution is obtained, after running 30 trials which gives the minimum number of PMUs and maximum measurement redundancy for normal operating conditions of the system.

The proposed method offers the optimum solution for IEEE 14 bus system as in Table XII. It shows the efficiency of zero injection bus constraint in MOPP problem. It further reduces the number of PMUs required for complete system observability.

The optimal number of PMUs and their locations, observability area of each PMU, are shown in Fig. 6 for IEEE 14 bus system.

Table XIII shows the different solutions of PMUs set for IEEE 30 bus system. It is shown that numbers of buses are observed more than a time is higher in the first PMUs set of Table XIII than any other sets. So, it is the best solution according to optimal PMU objective.

Table XIV shows the different solutions of PMUs set for IEEE 57 bus system. It is shown that numbers of buses are observed more than a time is higher in the first PMUs set of Table XIV than any other sets. Therefore, it is the best solution according to optimal PMU objective.

Table XV shows the different solutions of PMUs set for IEEE 118 bus system. It is shown that numbers of buses are observed more than a time is higher in the first PMUs set of Table 15 than any other sets. Hence, it is the best solution according to optimal PMU objective.

The optimal locations are determined in such a way that which PMUs set offers the minimum number of PMUs and maximum measurement redundancy for complete system observability, in normal power operating conditions, with zero injection bus consideration. Here, maximizing bus measurement redundancy is considered, randomly on the buses. Therefore, total system observability is improved.

The results presented in Table XVI, guarantee the complete observability of the system with minimum number of PMUs and the corresponding maximum measurement redundancy. It is clear that the total number of PMUs is considerably reduced for complete observability due to the inclusion of zero injection bus constraint and shown in Table XVII.

TABLE XII. SOLUTION FOR IEEE 14 BUS- CASE 2

Min. no. of PMUs	Locations of PMUs, (*P*)	*TSORI*	*BOI*	No. buses observed more than a time
3	2, 6, 9	16	1.14	1

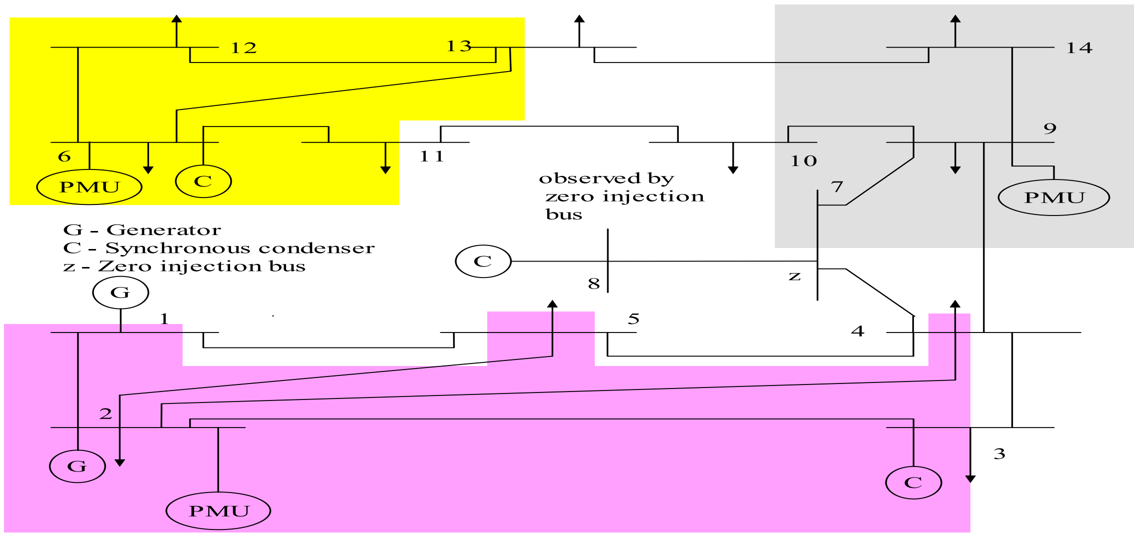

Figure 6. IEEE 14 bus system with PMU placement - Case 2

TABLE XIII. DIFFERENT SOLUTIONS FOR IEEE 30 BUS- CASE 2

Solution	Min. no. of PMUs	Locations of PMUs, (*P*)	*TSORI*	*BOI*	No. buses observed more than a time
1	7	2, 4, 10, 12, 15, 18, 27	41	1.36	7
2	7	1, 5, 10, 12, 18, 23, 27	36	1.2	2
3	7	1, 5, 10, 12, 18, 24, 27	37	1.23	3
4	7	1, 2, 10, 12, 18, 24, 27	39	1.3	6
5	7	2, 3, 10, 12, 19, 24, 27	36	1.2	5
6	7	3, 7, 10, 12, 18, 24, 27	40	1.3	3
7	7	3, 5, 10, 12, 18, 24, 27	37	1.23	3
8	7	3, 5, 10, 12, 18, 23, 27	36	1.2	2
9	7	1, 5, 10, 12, 19, 24, 27	37	1.23	4

TABLE XIV. DIFFERENT SOLUTIONS FOR IEEE 57 BUS- CASE 2

Solution	Min. no. PMUs	Locations of PMUs, (*P*)	*TSORI*	*BOI*	No. buses observed more than a time
1	11	1, 6, 13, 19, 25, 29, 32, 38, 41, 51, 54	57	1	3
2	11	1, 4, 13, 19, 25, 29, 32, 38, 41, 51, 54	58	1.01	2
3	11	1, 6, 13, 19, 25, 29, 32, 38, 51, 54, 56	58	1.01	2
4	11	1, 4, 13, 19, 25, 29, 32, 38, 51, 54, 56	58	1.01	2
5	11	1, 4, 13, 20, 25, 29, 32, 38, 51, 54, 56	59	1.03	2

TABLE XV. DIFFERENT SOLUTIONS FOR IEEE 118 BUS- CASE 2

Solution	Min. no. PMUs	Locations of PMUs, (*P*)	*TSORI*	*BOI*	No. buses observed more than a time
1	28	3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110	156	1.32	24
2	28	3, 8, 11, 12, 17, 20, 23, 29, 34, 37, 40, 45, 49, 53, 56 62, 73, 75, 77, 80, 85, 86, 91, 94, 101, 105, 110, 115	151	1.27	19
3	28	3, 8, 11, 12, 19, 22, 27, 31, 32, 34, 37, 40, 45, 49, 53 56, 62, 73, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110	151	1.27	17
4	28	3, 8, 11, 12, 17, 20, 23, 29, 34, 37, 40, 45, 49, 52, 56 62, 71, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110, 115	152	1.28	22
5	28	3, 8, 11, 12, 19, 21, 27, 31, 32, 34, 37, 42, 45, 49, 52 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110	153	1.29	20

TABLE XVI. OPTIMAL SOLUTIONS- CASE 2

IEEE system	Min. no. PMUs	Locations of PMUs, (*P*)	n_{pmu}/N, %	*TSORI*
14 bus	3	2, 6, 9	21%	16
30 bus	7	2, 4, 10, 12, 15, 18, 27	23 %	41
57 bus	11	1, 4, 13, 20, 25, 29, 32, 38, 51, 54, 56	19 %	59
118 bus	28	3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110	24 %	156

TABLE XVII. EFFICIENCY OF THE ZERO INJECTION BUS CONSTRAINT- CASE 2

IEEE system	Min. no. PMUs		No. PMUs are reduced
	Case 1	Case 2	
14 bus	4	3	1
30 bus	10	7	3
57 bus	17	11	6
118 bus	32	28	4

TABLE XVIII. COMPARISON OF PROPOSED RESULTS- CASE 2

Method	Min. no. PMUs required for IEEE system			
	14 bus	30 bus	57 bus	118 bus
Proposed	3	7	11	28
GA, [10]	3	7	12	29
IGA, [11]	3	7	11	29
BPSO, [12]	3	7	11	28
CLA, [31]	3	7	11	29
ILP, [32]	3	7	11	28
BICA, [34]	3	7	11	28
RTS, [35]	3	7	11	28
Dual search and SA, [36]	3	-	-	29
BILP, [37]	3	-	14	29

TABLE XIX. COMPARISON OF AVERAGE BUS MEASUREMENT REDUNDANCY- CASE 2

Method	Average bus measurement redundancy, (*TSORI/N*)			
	14 bus	30 bus	57 bus	118 bus
Proposed	1.143	1.467	1.105	1.322
IGA, [11]	1.143	1.167	1.035	-
CLA, [31]	1.143	1.467	1.105	-
BIP, [33]	1.143	1.367	1.035	1.322
BICA, [34]	1.143	1.367	1.035	1.322

TABLE XX. COMPARISON OF COMPUTATION EFFICIENCY - CASE 2

Method	Average execution time in seconds			
	14 bus	30 bus	57 bus	118 bus
Proposed	40	54	303	915
Modified BPSO, [12]	60	360	2580	5100

Table XVIII shows the comparison of proposed method with earlier methods. It shows that the proposed method is able to make a better MOPP solution.

This proposed method demonstrates the maximum measurement redundancy with the same minimum number of PMUs obtained in the available literatures, as shown in Table XIX. It should be noted that the obtained results are equal to the measurement redundancy reported in the available literatures in [31] and greater than in [11, 33, 34] for IEEE 30 and 57 bus system.

It is clear that, it can be proved that characteristic of ABC is computationally effective in offering better quality solutions obtained in minimum computation time at par with BPSO as shown in Table XX.

D. Case 3- MOPP problem for single line loss

Single line loss is a most common fault in the power systems. If two PMUs are observing a bus, then a related line loss will not affect the complete system observability. This can be modeled by modifying the constraints given as in Eq. (25),

$$c_i * P \geq 2 \tag{25}$$

Subject to the constraints from Eqs. (3-14), where c_i, the binary variable elements of the ith row of connectivity matrix J.

The optimal number of PMUs and their locations for complete system observability for the single line loss condition are presented in Table XXI.

Table XXII presents a comparison of the obtained minimum number of PMUs with other methods. In all cases, the results are either better than other methods or the same.

It is clear that, it can be proved that characteristic of ABC is computationally effective in offering better quality solutions obtained in minimum computation time when it is compared with BPSO for single line loss condition, as shown in Table XXIII.

Thus, the proposed method proved the efficiency in determining the optimal number of PMUs and their locations to maximize the observability and measurement redundancy for different operating conditions of the power systems, simultaneously.

TABLE XXI. OPTIMAL SOLUTIONS- CASE 3

IEEE system	Min. no. PMUs	Locations of PMUs, (P)	n_{pmu}/N, %	TSORI	BOI
14 bus	7	2, 4, 5, 6, 9, 10, 13	50 %	34	2.28
30 bus	11	2, 3, 6, 7, 10, 12, 15, 16, 19, 24, 29	43 %	52	1.73
57 bus	19	1, 3, 6, 12, 14, 15, 19, 27, 29, 30, 32, 33, 38 41, 49 51, 53, 55, 56	33 %	86	1.51
118 bus	53	1, 6, 10, 11, 12, 15, 17, 19, 21, 23, 24, 25, 27, 29 32 34, 35, 40, 42, 44, 46, 49, 51, 53, 56, 57, 59, 62 66 70, 73, 75, 76, 78, 80, 83, 85, 87, 89, 91, 92, 94 96 100, 102, 105, 106, 109, 111, 112, 115, 116, 117	45 %	242	2.06

TABLE XXII. COMPARISON OF PROPOSED METHOD- CASE 3

IEEE system	Proposed method		BPSO, [12]		BIP, [33]		BICA, [34]	
	Min. no. PMUs	TSORI	Min. no. PMUs	TSORI	Min. no. PMUs	TSORI	Min. no. PMUs	TSORI
14 bus	7	34	7	-	7	25	7	34
30 bus	11	52	15	-	13	50	11	52
57 bus	19	86	22	-	19	86	19	86
118 bus	53	242	62	-	53	242	53	242

TABLE XXIII. COMPARISON OF COMPUTATION EFFICIENCY- CASE 3

Method	Average execution time in seconds			
	14 bus	30 bus	57 bus	118 bus
Proposed	43	68	322	971
Modified BPSO, [12]	240	840	4800	13500

VII. CONCLUSION

This paper presented the application of binary coded artificial bee colony (ABC) algorithm for solving the Multi-objective OPP (MOPP) problems. The following points summarize the proposed work.

ABC algorithm, when applied to the MOPP problem for normal operating conditions, is able to offer the optimal number of PMUs with their locations and maximum measurement redundancy of the systems, when compared with other conventional and non conventional techniques, reported in the earlier literatures.

Multi-objective PMU placement problem has also been proposed for single line loss in the power systems and it is solved using ABC algorithm. From the comparative analysis, it is observed that the proposed methodology is efficient in determining the optimal number of PMUs and their locations under normal and contingency condition in order to maximize the observability and measurement redundancy of the systems, simultaneously. The feasibility and performance of the proposed methodology are demonstrated on IEEE 14, 30, 57 and 118 bus systems. The results presented in this paper will encourage the researchers in using an ABC algorithm for larger power systems.

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