

# Novel Approach for Electromagnetic Actuators Analysis in Transient Behavior

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**Abstract**—A new model of the actuator is proposed in this paper. It considers the nonlinear electromagnetic phenomena in the ferromagnetic core, as well as the influence of the mechanical load during the plunger movement. According to our approach, the entire system that includes the magnetic circuit, the electric circuit and the mechanical parts is mathematically modeled through a differential algebraic equation system (DAE). Therefore, a corresponding analog nonlinear electric circuit described by a similar mathematical model is conceived and implemented in an electric circuit simulation program capable to analyze its behavior in steady state or dynamic regimes. The SPICE simulator has been chosen as implementation platform and a case study has been performed to prove the feasibility and efficiency of our approach. The simulation result contains electromagnetic and mechanical quantities that were represented as time-domain functions. The method is remarkable through an extremely short computation time when compared with the classical methods based on the discretization of the domain.

**Index Terms**—actuators, circuit simulation, eddy currents, equivalent circuits, nonlinear systems.

## I. INTRODUCTION

As main components of the mechanical switching devices (e.g. electromagnetic contactors and relays) the electromagnets are widely used for remote controls and control systems [1]. Even if the worldwide trend is to replace the mechanical switching with static (electronic) switching, many devices and systems with actuators are still designed and manufactured for different applications. Thus the study of electromagnets continues to interest the scientific community that tries to solve a lot of problems related to their operation.

The operation of AC electromagnets in steady state is mainly of a theoretical interest. That is why it is studied especially to verify some new hypothesis or analysis methods [1]–[4]. Consequently the researchers concentrate their interest on the more complex problem related to the dynamic behavior of the electromagnets. Many research groups focused their efforts on the study, analysis and improvement of the electromagnetic actuators characteristics, aiming the design optimization. The dynamic behavior is of most importance, but it is difficult to be predicted because of the system nonlinearity. In general in order to solve electromagnetic problems one uses commercial software programs, as QuickField or FEMM, that allow the coupling between the electromagnetic field

and the supply circuit of the device. More complex software programs, as ANSYS, permit also a mechanical analysis of the equipment in order to determine the mechanical and/or electromagnetic stresses. Many authors used finite-element method (FEM) or finite-difference method (FDTD) based models to solve this problem, and implemented them in their own computing programs [5]–[12]. It is notable the approach described in [5], where a complex model coupling the electrical, magnetic and mechanical subsystems into the FEM program is proposed or the simplified one-dimensional model from [6]. Also parametric or axisymmetric device models were considered in [8]–[12]. In many works, the authors' attention was focused on the magnetic fluxes produced inside the electromagnets, but they neglected the influence of the mechanical part of the devices.

Most of these software programs require appropriate hardware resources to operate properly. And yet they have often needed tens of hours to finish a simulation. This represents a major inconvenience especially in the design and optimization. Modeling and simulation of the electromagnetic devices based on equivalent analogical circuits largely solve this inconvenience. By choosing an appropriate model of the device one can detail those aspects of interest in its operation. Moreover an equivalent electric circuit based model makes possible its integration in complex circuits that could include static converters.

Our paper proposes a new model of electromagnetic actuators applied to AC plunger-type magnets. It has the main advantage of modeling the entire electromagnetic system (including electric, magnetic and mechanical components) using electric circuit elements only. We took into account the nonlinear properties of magnetic cores, the effect of the eddy currents, but also the time varying influence of the mechanical load. The developed model is easy to be implemented in any commercial program for electric circuit analysis, requiring reasonable hardware resources and computing effort. Finally the SPICE software has been chosen as simulation tool.

## II. PRINCIPLES OF THE ACTUATOR MODELING

The main structure of the analyzed plunger-type magnet is presented in Fig. 1.

This type of electromagnet is one of the most used in common practice (e.g. in contactors). It consists in a movable core with a rectangular cross section (plunger), a fixed core, a coil supplied by an AC voltage and a short-circuit ring. This latter is essential to improve the electromagnet operation [1].

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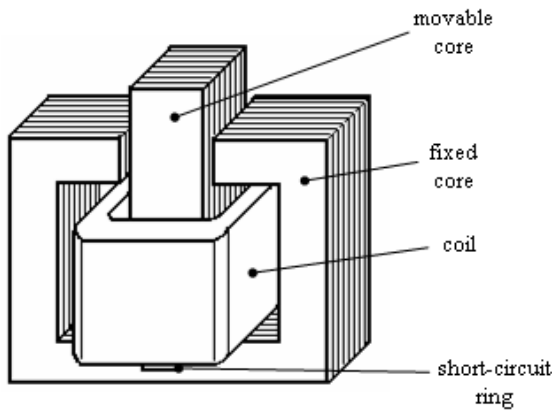


Figure 1. AC plunger-type magnet.

The actuator model joins the model of the ferromagnetic core as part of the magnetic circuit, the electrical circuit and the mechanical phenomena. These main components of the model are presented here.

A new high precision model for nonlinear ferromagnetic cores was considered for the AC electromagnet. It takes into account the nonlinear phenomena and the eddy current effect. These influences are of major importance in the transient behavior of the electromagnet. The core model was presented in detail in [13]. A scheme of principle is shown in Fig.2.

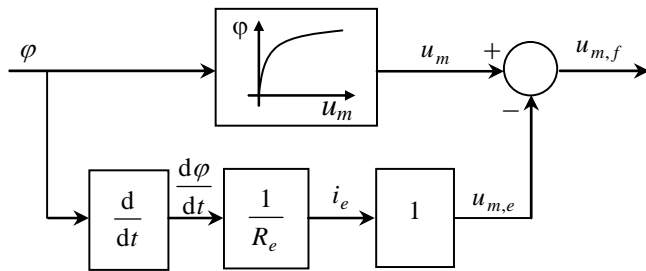


Figure 2. Block diagram of a ferromagnetic piece model that considers the eddy current losses and the reaction magnetic field of eddy currents.

This diagram was realized based on the mathematical model associated to the piece of ferromagnetic core. It considers the magnetization curve and the eddy currents. The equations of the model are:

$$u_m = f(\varphi) \quad (1)$$

$$i_e = \frac{1}{R_e} \cdot \frac{d\varphi}{dt} \quad (2)$$

$$u_{m,e} = i_e \quad (3)$$

$$u_{m,f} = u_m - u_{m,e} \quad (4)$$

In the above relations  $\varphi$  is the magnetic flux through the ferromagnetic piece; it represents the input variable of the model.  $u_m$  is the component of the magnetic voltage drop that does not depend on eddy current.  $i_e$  is the equivalent eddy current. According to the Ampere's law, the magnetic voltage drop produced by the equivalent eddy current  $u_{m,e}$  (a parasitic component of the final magnetic voltage drop) is equal to the eddy current  $i_e$ . The magnetic voltage drop  $u_{m,e}$  is opposite to the component  $u_m$  and thus these two magnetic voltages determine the final magnetic voltage drop

$u_{m,f}$  in any moment of time.  $u_m$ ,  $i_e$ ,  $u_{m,e}$  and  $u_{m,f}$  are the variables of the system (1) – (4).

The model presented here corresponds to a ferromagnetic piece of constant section and a certain length. In the circuit equivalent model, the  $\varphi - u_m$  characteristic (corresponding to the first-magnetization curve of the ferromagnetic material [13]) can be modeled through a voltage-controlled nonlinear resistance. The other blocks from the lower side of Fig. 2 build a linear network having the role of considering the eddy currents effect. The losses produced by these currents lead to Joule effects and reaction magnetic fluxes that modify the magnetization curve in dynamic regimes.

We considered an equivalent eddy current  $i_e$  that has the same effect as the real current through the ferromagnetic piece. It is calculated as function of magnetic flux derivative and an equivalent resistance passed by the eddy current,  $R_e$ . The equivalent resistance of a ferromagnetic piece is constant; it depends on the electric conductivity (the conductivity can be computed in terms of specific power losses  $p_{Fe}$  specified by the manufacturer for a certain reference frequency of the magnetic field, assumed as sinusoidal, and a certain peak value of the magnetic flux density), on the geometrical dimensions of the magnetic piece and on the material mass density [13].

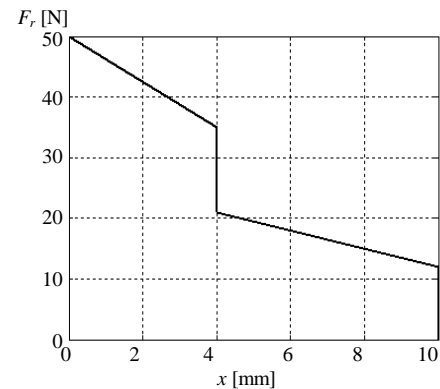
The block diagram presented in Fig. 2 can be entirely modeled as an equivalent circuit that includes controlled sources [13].

The electric circuit accomplishes the voltage equation:

$$u(t) = U_{\max} \sin(\omega t + \alpha) = R \cdot i(t) + N \frac{d\varphi(t)}{dt}. \quad (5)$$

$u(t)$  represents the sinusoidal supply voltage,  $R$  is the resistance of the coil,  $N$  is the number of the coil turns and  $i(t)$  is the current in the supply circuit. The inductive term  $d\varphi/dt$  is imposed by the actual time-dependent magnetic flux. In an equivalent circuit model this derivative can be obtained from the magnetic flux by using two controlled sources and a unity-inductor [13].

The mechanical behavior of the system is influenced by the characteristic of the resistance force given in Fig.3.

Figure 3. Nonlinear mechanical load  $F_r(x)$ .

While the central gap length  $x$  decreases, this force depends on the weight of the mobile part and the friction (here, the vertical section from  $x = 10$  mm), on the return springs and auxiliary contacts forces (linear increasing section) and on the force of the main contact strings (here the second vertical section from  $x = 4$  mm and then the

increasing one) [1].

The active magnetic force  $F_m$  is calculated based on the Maxwell stress theory [14] applied to the surface related to the movable core S:

$$F_m = \int_S T_m \cdot dA = \frac{S}{2} (B_{air} \cdot H_{air} - B_{Fe} \cdot H_{Fe}) \quad (6)$$

Because the air is a linear medium, (6) can be written as [15]:

$$F_m = \frac{\varphi^2}{2\mu_0 S} - \frac{S}{2} B_{Fe} H_{Fe} \quad (7)$$

The equation that connects the magnetic and the mechanical parts of the electromagnetic system includes the magnetic force  $F_m$ , the resistance (opposite) force  $F_r$  and the inertial force. The inertial force depends on the mass of the armature  $m$  and on the acceleration  $a$ . The position of the armature in its movement  $x$  is used instead of the acceleration, so that:

$$F_m = m \cdot a + F_r = m \cdot \frac{d^2 x}{dt^2} + F_r \quad (8)$$

All the considerations presented above are included in the electromagnet model.

### III. MODEL IMPLEMENTATION AND SYSTEM ANALYSIS

Our purpose was to realize a dynamic model for the actuator based on the precision model presented above. It combines magnetic, electric and mechanic components. We choose to simulate and to analyze an AC electromagnet of 380V/50 Hz that is part of an AC contactor.

The parameters that are necessary to our analysis refer to the resistances of the coil and of the short-circuit ring, the number of the coil turns and the dimensions and properties of the magnetic cores. These data were calculated based on the algorithms presented in the literature [1] and were introduced in our model. Also the  $B-H$  magnetization curve and the nonlinear characteristic of the resistance force as function of the gap must be known. The magnetization curve corresponds to the silicon steel sheets of 0.35 mm [1], [4], while the resistance force – gap dependency was represented in Fig. 3.

The AC electromagnet model is based on the equivalent model of the magnetic core and on the coupling equations presented above. This model is illustrated in Figs. 4–6.

Fig. 4 (a) represents the supply electric circuit. Here the supply voltage, the network parameters and the coil resistance are considered as well as the influence of the magnetic circuit through the voltage controlled voltage source that introduces the induced voltage  $e$ . Fig. 4 (b) corresponds to the electric circuit of the short-circuit ring, where an induced current appears. Here the ring resistance  $R_{ring}$  was considered as well as the influence of the magnetic flux that crosses the ring,  $\varphi_{ring}$ , by means of two controlled sources. Both the coil and the ring currents are considered in the equivalent magnetic circuit from Fig. 5.

The diagram of Fig. 5 corresponds to the equivalent magnetic circuit. The reluctances are modeled using equivalent electric resistances. For the nonlinear reluctances of the cores the model of ferromagnetic piece that was explained above was used. The linear reluctances correspond

to the gap or to the leakage (dispersion) reluctances. The current controlled voltage sources have the role of magnetomotive forces (in case of the coil and of the short-circuit ring respectively). Because the plunger is moving, the reluctances of the movable core  $R_{FE\_1}$  and of the central gap  $R_\delta$  are variable depending on the stroke  $x$ . By using this equivalent circuit, various magnetic fluxes, as magnetic flux in the plunger  $\varphi$  and the magnetic flux through the ring  $\varphi_{ring}$  can be computed and analyzed.

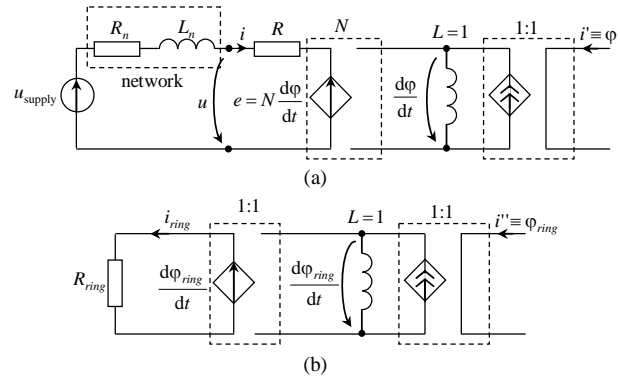


Figure 4. Model of the actuator electric parts.

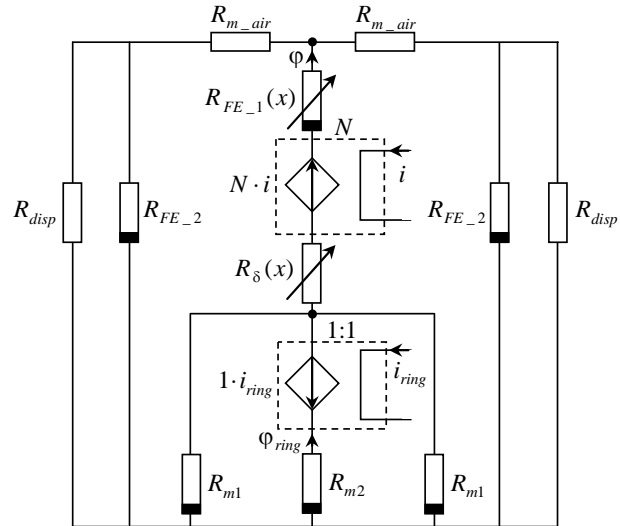


Figure 5. Model of the actuator magnetic core.

Fig. 6 shows the block diagram of the mechanical part. The magnetic flux through the plunger is used to calculate the magnetic force based on relation (7). Thus two current controlled voltage sources were used to transform the quantities into electric potentials. Then some “multiply” and “sum” blocks are used to multiply two quantities with a given constant K or to subtract one component of the magnetic force from the other. Also a “table” block was inserted for the  $B-H$  nonlinear dependency. Then the inertial force  $F_a$  is obtained from the magnetic and resistance forces through a subtraction. Using the relation (8) we can obtain the acceleration  $a$  and then the speed  $v$  and the stroke  $x$  by means of the integration blocks. In our mechanical model the integration blocks were realized with unity-inductors, similar into [13]. The resulted stroke  $x$  influences not only the value of the resistance force (Fig.3), but also the values of the variable reluctances from Fig. 5.

Thus the equivalent circuits of Figs. 4, 5 and 6 are interconnected through the magnetic flux of the plunger, the coil current, the ring current and the plunger's position respectively, by means of controlled sources.

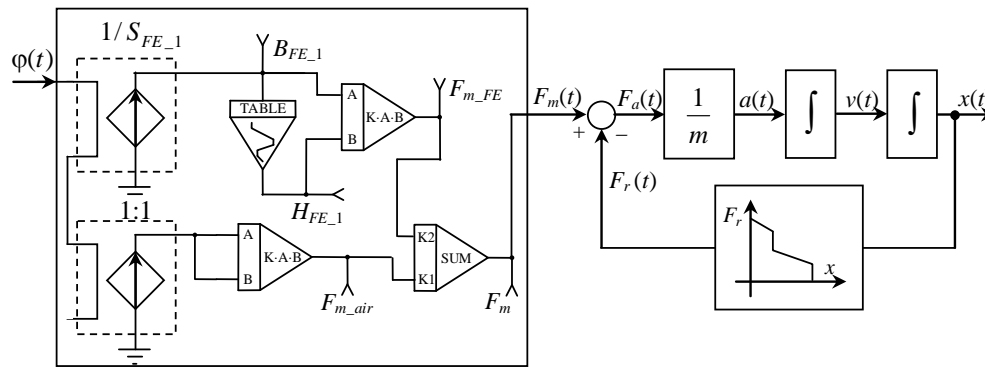


Figure 6. Block model of the mechanical part of the actuator.

#### IV. RESULTS

The model presented above was implemented in SPICE [16],[17] with the aim of analyzing the dynamic behavior corresponding to the connecting operation. By solving the equivalent circuit, the waveforms of several electromagnetic and mechanic quantities were obtained. Some of them are presented below.

Fig. 7 presents the waveforms of the supply voltage and of the coil current during the dynamic regime caused by the connection. The connection process started at  $t = 0$  ms.

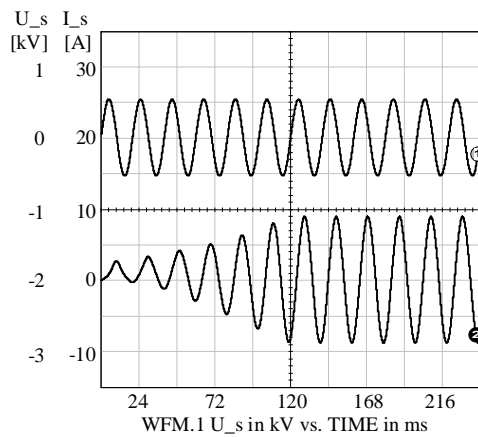


Figure 7. Supply voltage (up) and coil current (down) during the connecting operation.

Fig. 8 shows the magnetic flux inside the plunger during its movement and after it stops. In the analyzed case one can notice that the steady state regime is reached after 120 ms (after the free components damping).

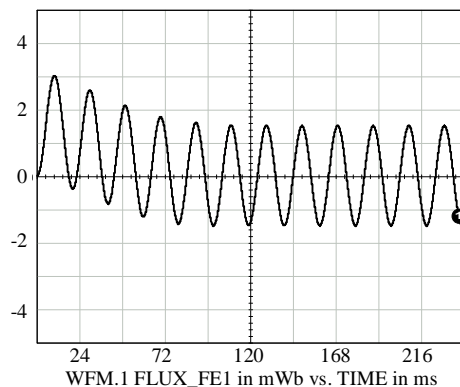


Figure 8. Magnetic flux inside the movable core during the connecting operation

Fig. 9 shows the magnetization cycle of the movable core

(magnetic flux  $\phi$  versus magnetic voltage drop) during the dynamic regime. One notices that the saturation level is reached during the movement, so that the electromagnetic quantities have distorted waveforms.

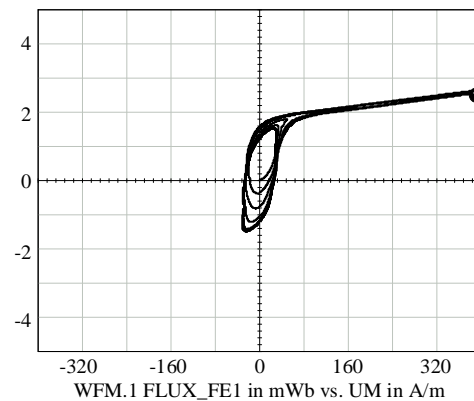


Figure 9. Magnetization cycle of the movable core during the dynamic regime.

Fig. 10 shows the current through the short-circuit ring during and after the connection. One can see that, when the steady-state regime is achieved (after 120 ms), the current through the short-circuit ring is distorted and displaced in phase by  $-90^\circ$  from the magnetic flux and from the control coil current. This is in accordance with the electromagnetic field theory.

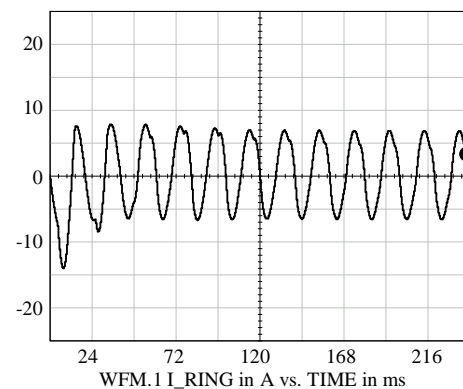


Figure 10. Current waveform in the short-circuit ring during the dynamic regime.

Also with respect to Fig. 11 (the steady-state current through the short-circuit ring) one remarks the distortion of the current. By using the facilities of SPICE software the Fast Fourier Transform (FFT) analysis can be done. Thus we notice the appearance of the 3rd, 5th and 7th order harmonics.

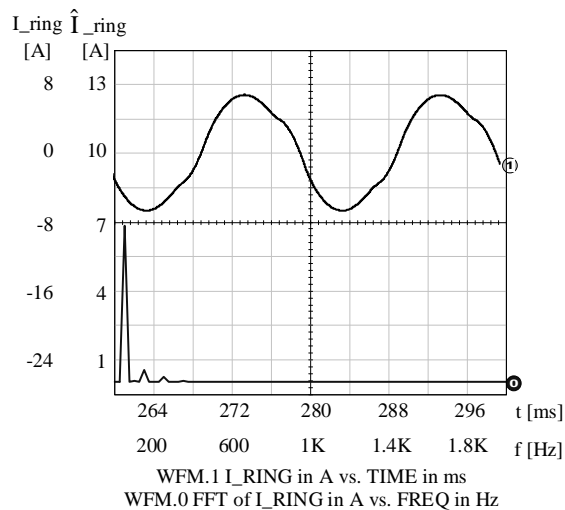


Figure 11. FFT analysis of the current in the short-circuit ring, after the connection.

Figs. 12 and 13 show the time variation of the magnetic and resistance forces, during and after the connection. After the connection the magnetic force has a positive component that maintains constantly the position of the plunger. One can notice the similarity between the  $F_r$  shape from Fig. 3 and those from Fig. 13, where the variation of the resistance force  $F_r$  while the plunger is moving (in time) is illustrated.

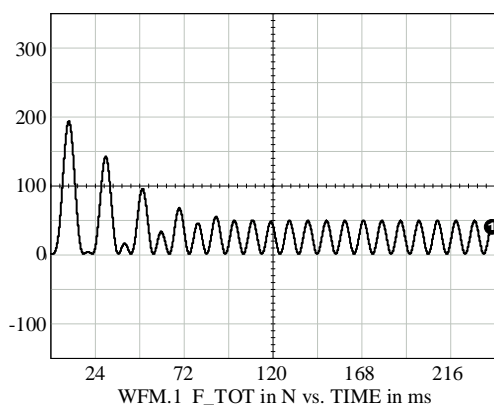


Figure 12. Variation of the magnetic force  $F_m$  during the connecting operation

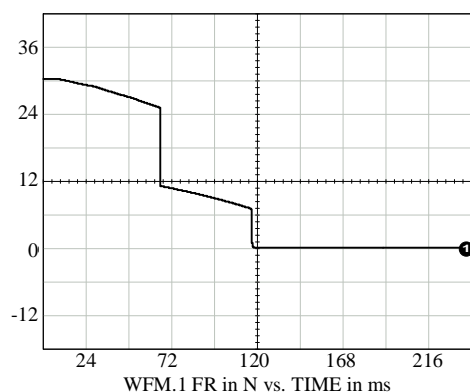


Figure 13. Variation of the resistant force  $F_r$  during the connecting operation.

The time variation of the plunger position during the connection is shown in Fig. 14. The first part of the characteristic is nonlinear due to the time variation of the speed (depending on acceleration), as it is shown in Fig. 15. The simulation results show that the movement ends after 120 ms, when the speed becomes zero and the position remains constant at 10 mm. For this reason Fig. 15 shows

the speed variation only during the movement of the core.

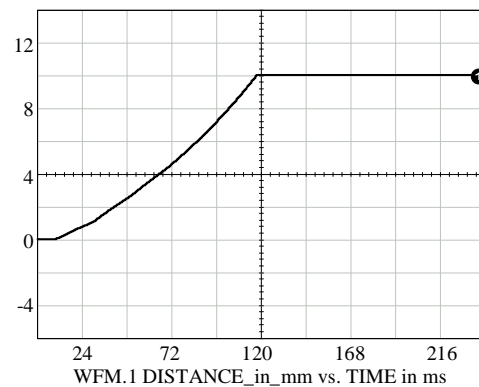


Figure 14. Variation of the movable core position during the dynamic regime.

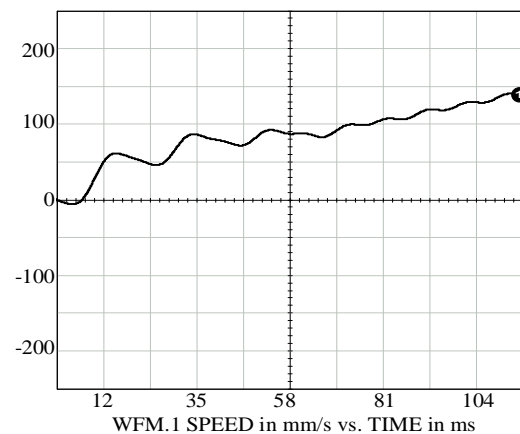


Figure 15. Speed variation during the movement of the core.

In order to validate our equivalent circuit model we chose to solve the same problem using a commercial software for electromagnetic analysis. The FLUX 2D/3D package was used for this purpose. Based on a dynamic regime analysis using the finite element method we obtained data concerning the magnetic field distribution in the actuator and the time variation of certain electric quantities (supply voltage and coil current) and mechanical quantities (forces, movable core speed, etc.). Some of these results are presented below.

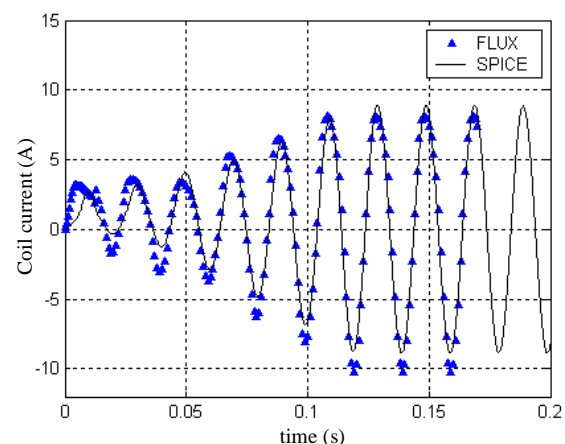


Figure 16. Coil current variation during the connecting operation; comparative presentation of the results obtained using the equivalent circuit model (SPICE) and a commercial software for electromagnetic analysis (FLUX).

Fig. 16 presents in a comparative manner the time variation of the coil current during the connecting operation obtained using the circuit model (that was implemented in

SPICE) and FLUX. We can notice that the two waveforms are almost identical. The small differences between them could be caused by the different convergence techniques that are used by the two programs.

Fig. 17 shows the time variations of the magnetic force produced by the actuator that were obtained using the same two programs. The resulted waveforms are very close, too. The differences between them can be considered as a result of the differences obtained in the current variations.

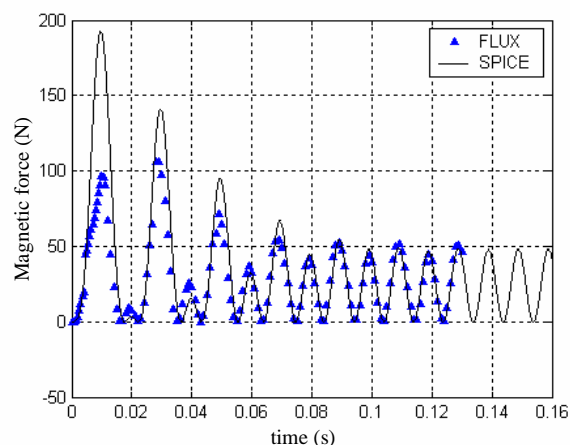


Figure 17. Magnetic force variation during the connecting operation; comparative presentation of the results obtained using the equivalent circuit model (SPICE) and a commercial software for electromagnetic analysis (FLUX).

The results were compared qualitatively also using data offered by the technical literature [1], [5]–[7], [10], [18]–[20]. We observed good similarities between them.

The entire comparative analysis confirmed our model validity. This recommends it for solving some problems related to the actuator operation.

We must emphasize that even if our proposed model does not specify the magnetic field distribution inside the actuator (by comparing with a field analysis), it has the advantage of offering in a shorter time results concerning the time variation of the ring current, of the magnetic flux through the movable core or the  $\varphi$ – $u_m$  dependency in dynamic regimes, some of them being impossible to be obtained using other programs based on field analysis.

## V. CONCLUSION

The paper proposed a new model for the actuators that operate in dynamic regimes. It considers both the nonlinear electromagnetic phenomena in the ferromagnetic core and the nonlinear behavior of the mechanical load.

The model permits computing and judging several electromagnetic and mechanical quantities. It represents a step forward in the time-domain simulation of the electromagnets, by bringing together the electromagnetic and mechanical equations that describe their operation and transposing them into a complex equivalent electric circuit. In this manner the system is easy to be implemented using any circuit analysis and simulation program, as useful tool for research and design activity.

The basic elements used in the actuator model can be used afterwards to model other electromagnetic devices as transformers or electric motors especially in those applications that use static converters.

## REFERENCES

- [1] G. Hortopan, G. Cosmin, M. Huhulescu, V. Panaite, D. Simulescu, R. Tomoioga, *Low Voltage Electrical Apparatus (Aparate electrice de joasa tensiune)*, Ed. Tehnica, Bucharest, 1969, p.126-144, 227-324.
- [2] S. Yamada, K. Bessho, "Harmonic field calculation by the combination of finite element analysis and harmonic balance method", *IEEE Trans. Magn.*, vol. 24, no. 6, pp. 2588-2590, November 1988. [Online]. Available: <http://dx.doi.org/10.1109/20.92182>
- [3] G. A. Cividjian, N. G. Silvis-Cividjian, A. G. Cividjian, "Inductance of a plunger-type magnet", *IEEE Trans. Magn.*, vol. 34, no. 5, pp. 3685-3688, September 1998. [Online]. Available: <http://dx.doi.org/10.1109/20.718529>
- [4] I. G. Sirbu, M. Iordache, L. Mandache, "A new model for ac plunger-type magnets in steady-state regime", *Annals of the University of Craiova - Electrical Engineering Series*, nr. 34, vol.1, pp.19-24, 2010.
- [5] R. Gollee, G. Gerlach, "An FEM-based method for analysis of the dynamic behavior of ac contactors", *IEEE Trans. Magn.*, vol. 36, no. 4, pp. 1337-1340, July 2000. [Online]. Available: <http://dx.doi.org/10.1109/20.877686>
- [6] B. Lequesne, "Dynamic model of solenoids under impact excitation, including motion and eddy currents", *IEEE Trans. Magn.*, vol. 26, no. 2, pp. 1107-1116, March 1990. [Online]. Available: <http://dx.doi.org/10.1109/20.106512>
- [7] Y. Kawase, S. Ito, "Analysis of attractive force of pull-type single phase ac electromagnets", *IEEE Trans. Magn.*, vol. 26, no. 2, pp. 1046-1049, March 1990. [Online]. Available: <http://dx.doi.org/10.1109/20.106500>
- [8] J. R. Riba Ruiz, A. Garcia Espinosa, "A novel parametric model for ac contactors", *IEEE Trans. Magn.*, vol. 44, no. 9, pp. 2215-2218, September 2008. [Online]. Available: <http://dx.doi.org/10.1109/TMAG.2008.2000544>
- [9] O. Bottauscio, M. Chiampi, A. Manzin, "Diffusion and hysteresis in axisymmetric electromechanical devices", *IEEE Trans. Magn.*, vol. 39, no.2, pp. 990-997, March 2003. [Online]. Available: <http://dx.doi.org/10.1109/TMAG.2003.808585>
- [10] T. Kajima, "Dynamic model of the plunger type solenoids at deenergizing state", *IEEE Trans. Magn.*, vol. 31, no.3, pp. 2315-2323, May 1995. [Online]. Available: <http://dx.doi.org/10.1109/20.376228>
- [11] R. E. Clark, G. W. Jewell, D. Howe, "Dynamic modeling of tubular moving - magnet linear actuators", *Journal of Applied Physics*, vol. 93, no. 10, pp. 8787-8790, May 2003. [Online]. Available: <http://dx.doi.org/10.1063/1.1544518>
- [12] P. Lombard, G. Meunier, "A general purpose method for electric and magnetic combined problems for 2D, axisymmetric and transient systems", *IEEE Trans. Magn.*, vol. 29, no.2, pp. 1737-1740, March 1993. [Online]. Available: <http://dx.doi.org/10.1109/20.250741>
- [13] L. Mandache, K. Al-Haddad, "An accurate design tool for filter inductors", in *Proc. of the 33rd Annual Conference of the IEEE Industrial Electronics Society - IECON 2007*, Taipei, Taiwan, 2007, pp. 1414-1419.
- [14] C. I. Mocanu, *Electrotechnics Fundamentals. Electromagnetic Field Theory (Bazele electrotehnicii. Teoria cimpului electromagnetic)*, Ed. Didactica si Pedagogica, Bucharest, 1991, p.497-511.
- [15] I. G. Sirbu, M. Iordache, L. Mandache, "Considerations on the simulation models of ac plunger-type magnets", in *Proc. of the 7th International Symposium on Advanced Topics in Electrical Engineering ATEE - 2011*, Bucharest, Romania, 2011, pp.25-30.
- [16] H. G. Brachtendorf, C. Eck, R. Laur, "Macromodeling of hysteresis phenomena with SPICE", *IEEE Trans. On Circuits and Systems-II: Analog and Digital Signal Processing*, vol. 44, no. 5, pp. 378-388, May 1997. [Online]. Available: <http://dx.doi.org/10.1109/82.580845>
- [17] P. W. Tuinenga, *SPICE, A Guide to Circuit Simulation & Analysis Using PSpice*, Prentice Hall, Englewood Cliffs, New Jersey, 1988.
- [18] Y. Kawase, S. Tatsuoka, T. Yamaguchi, "3-D finite element analysis of operating characteristics of ac electromagnetic contactors", *IEEE Trans. Magn.*, vol. 30, no.5, pp. 3244-3247, September 1994. [Online]. Available: <http://dx.doi.org/10.1109/20.312629>
- [19] Y. Kawase, O. Miyatani, T. Yamaguchi, "Numerical analysis of dynamic characteristics of electromagnets using 3-D finite element method with edge elements", *IEEE Trans. Magn.*, vol. 30, no.5, pp. 3248-3251, September 1994. [Online]. Available: <http://dx.doi.org/10.1109/20.312630>
- [20] O. Bottauscio, M. Chiampi, A. Manzin, "Advanced model for dynamic analysis of electromechanical devices", *IEEE Trans. Magn.*, vol. 41, no.1, pp. 36 - 46, January 2005. [Online]. Available: <http://dx.doi.org/10.1109/TMAG.2004.840367>