Bayes-Based Fault Discrimination in Wide Area **Backup Protection**

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Abstract-Multivariate statistical analysis is an effective tool to finish the fault location for electric power system. In Bayesian discriminant analysis as a subbranch, by the research of several populations, one can calculate the conditional probability that some samples belong to these populations, and compare the corresponding probability. The sample will be classified as population with maximum probability. In this paper, based on Bayesian discriminant analysis principle, a great number of simulation examples have confirmed that the results of Bayesian fault discriminant in wide area backup protection are accurate and reliable.

Index Terms—bayesian discriminant analysis, fault discrimination, phasor measurement unit, PMU, wide area backup protection.

I. Introduction

With the gradually deregulation of power system and the widely introduction of competitive mechanism, the current power system devices are operated close to their physical limits day by day, and the power transmission grids are interconnected more and more tightly. In the circumstances, the relay protection, which always plays the role as the first defending line, will undertake more responsibility for security and stability of power system. Up to now, the principles of main protection have a well development in theoretical research and field inspection. However, the backup protection, as an efficient supplement for primary one, is still puzzled by a series of problems, such as complicated in cooperation relationship, long in operation time, difficult in setting and configuration, noneffective in identification of flow transferring and so on [1]-[6]. Therefore, in many worldwide blackouts the traditional backup protections have acted "correctly" according to the existing design philosophy of protection, but which actually added fuel to the fire in accelerating breakdown of region power system. It is the root cause that only the local information of power grid is used in the conventional backup protection, which is failed in adopting more valid status data from a more wide area in design and implementation.

Currently, the new measure system as WAMS/PMU (WAMS, Wide Area Measurement System; PMU, Phasor Measurement Unit) could be used to obtain the global operation condition information of power system [7]–[13]. In other words, the synchronized phasor data of nodal

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voltage and branch current with the same time stamp is becoming one new data source for power system analysis and design. In this way, the novel backup protection based on wide area synchronized information has attracted more attentions in field of power system relay protection in the past two decades. The basic rule of this novel protection is to locate the faulted components as soon as possible within the inherent time delay of backup protection, and in this locating process the synchronized phasor data uploaded by the widely installed PMUs is used for its short time refresh period. According to the fault location results, the corresponding tripping strategy could be schemed. Once the primary protection fails in acting, the backup one will opearte to isolate the related faulted components by tripping the schemed circuit breakers set. Hence, the fault location method is the one of key issues for novel wide area backup protection. Extracting fault feature information from large amount of synchronized data will be a new challenge for locating of faulted components or faulted area.

Multivariate statistical analysis theory is superior in reducing dimensions and simplifying data structure featured without key information loss. In order to code with the huge quantity of synchronized information supplied by WAMS, the multivariate statistical analysis theory could be used as a powerful mathematical tool. As to fault feature study in power system, reference papers [14]-[22] have provided a new way for introducing cluster analysis, pattern recognition and principal component analysis into the related fields of power system, especially for fault location needed in novel wide area backup protection, and the results of above methods are encouraged.

Cluster analysis based fault location method [14,15] focuses on classifying the changes of electrical quantities owing to the system failure. Then, the set of measure points (e.g. nodal voltage phasor in sequence or phase form) with most effected will be confirmed as fault section. In summary, this method could satisfy the rapid requirement of locating process, but in precision there is a further refinement step to find the specific abnormal component. In practice, this characteristic of coarse classification in this method will be widely used to determine the fault section as the first step.

The two discriminant analysis principles in pattern recognition, named Mahalanobis discriminant analysis and Fisher discriminant analysis, have been adopted in fault locating process in [16]-[17]. In this method, two discriminant functions corresponding to faulted component set and normal component set respectively are used to decide which group one specific component should belong to. Pattern recognition based algorithm has a good performance in accuracy of fault location. In addition, it is possible for this method to locating faulted component quickly after the combination with Cluster Analysis.

The above Cluster-Analysis-based method and Pattern-Recognition-based method do not reduce the dimensions of information matrix, therefore there may be a heavy calculation burden for large scale power grids. The principal component analysis (PCA) [18]–[22] is able to extract fault feature efficiently by finding the first component. Then the faulted component will be directly related to the biggest coefficient or biggest score of corresponding principal component. However scale of covariance matrix and complexity of principle component representation are sensitive with the node size of power system [23].

Different to Mahalanobis and Fisher discriminant analysis, the Bayes discriminant method can analyze probability of population appearance and losses caused by misjudgment simultaneously, and integrate the prior probability with posterior probability of research object systematically. Having taken the common fault types of power system into consideration, the Bayes-based method is proposed to give a partition for the membership of each element (healthy or faulted), in which the node status quantities are adopted as basic data provide by PMU.

The paper is organized as follows. In Section 2, the basic theory of Bayesian discriminant analysis is introduced. In Section 3, the novel Bayes-based faulted components location method is clarified in detail, in which wide area synchronized information is used as the input data. Finally, the paper is concluded in Section 4.

II. BAYESIAN DISCRIMINANT ANALYSIS PRINCIPLES

Bayesian statistics is an important branch of statistics. The basic idea of Bayesian statistics can be described as, suppose one has already had a certain understanding on the research objects before sampling, in which this kind of cognition is usually expressed as prior distribution, and then, the prior cognition can be modified according to samples, and posterior distribution can also be obtained. Based on the posterior distribution, various statistical inference can be carried through. Adopted Bayesian statistical idea into discriminant analysis, one can get Bayesian discriminant analysis. Let us illuminate the Bayesian discriminant principles of two populations [24, 25].

Suppose G_1, G_2 are two different p -dimensional populations which are known, their probability density functions are $f_1(x)$ and $f_2(x)$ respectively. The prior probability of G_1, G_2 appearance is:

$$p_1 = P(G_1), \quad p_2 = P(G_2)$$
 (1)

and $p_1 + p_2 = 1$.

For p-dimensional index observation $x = (x_1, x_2, \dots, x_p)^T$, its value space is p-dimensional Euclid space \mathbb{R}^p . In fact, a criterion is just a partition of \mathbb{R}^p , denoted as R_1, R_2 , and satisfy the following conditions:

$$R_1 \cup R_2 = \mathbb{R}^p, \qquad R_1 \cap R_2 = \emptyset$$
 (2)

A partition $R = (R_1, R_2)$ is equivalent to a criterion. Under the discriminant criterion R, the misjudgment probability of sample came from G_1 which is regarded as G_2 is,

$$P(2 | 1, R) = \int_{R_2} f_1(x) dx$$
 (3)

And the misjudgment probability of sample came from G_2 which is regarded as G_1 is,

$$P(1 \mid 2, R) = \int_{R} f_2(x) dx$$
 (4)

Suppose the misjudgment cost of sample came from G_1 which is regarded as G_2 is $c(2 \mid 1)$, and the misjudgment cost of sample came from G_2 is regarded as G_1 is $c(1 \mid 2)$. Generally speaking, the misjudgment cost of sample came from G_i is regarded as G_j can be expressed as $c(j \mid i)$, and $c(1 \mid 1) = c(2 \mid 2) = 0$. Bayesian discriminant analysis is just seeking for the partition $R = (R_1, R_2)$, which will minimize the average misjudgment cost.

For a new sample x, one can get the posterior probability of populations G_1, G_2 based on Bayesian principles,

$$\begin{cases}
P(G_1 \mid x) = \frac{p_1 f_1(x)}{p_1 f_1(x) + p_2 f_2(x)} \\
P(G_2 \mid x) = \frac{p_2 f_2(x)}{p_1 f_1(x) + p_2 f_2(x)}
\end{cases}$$
(5)

It can be proved, if c(2|1) = c(1|2), the optimization division of two populations based on Bayesian discriminant is,

$$\begin{cases} R_1 = \{x : P(G_1 \mid x) \ge P(G_2 \mid x)\} \\ R_2 = \{x : P(G_1 \mid x) < P(G_2 \mid x)\} \end{cases}$$
 (6)

So, one can obtain Bayesian discriminant criterion of two populations in the case of equivalent cost:

$$\begin{cases} x \in G_1, & \text{If } P(G_1 \mid x) \ge P(G_2 \mid x) \\ x \in G_2, & \text{If } P(G_1 \mid x) < P(G_2 \mid x) \end{cases}$$
 (7)

Meanwhile, the optimization division R will minimize the misjudgment probability, namely

$$p^* = p_1 P(2 \mid 1, R) + p_2 P(1 \mid 2, R)$$
 (8)

Actually, if c(2|1) = c(1|2) = c, then the average misjudgment cost is cp^* , so,

$$p^* = \int_{R_1} p_2 f_2(x) dx + \int_{R_2} p_1 f_1(x) dx$$

$$= \int_{R_1} p_2 f_2(x) dx - \int_{R_1} p_1 f_1(x) dx$$

$$+ \int_{R_1} p_1 f_1(x) dx + \int_{R_2} p_1 f_1(x) dx$$

$$= \int_{R_1} (p_2 f_2(x) - p_1 f_1(x)) dx + p_1$$
(9)

III. BAYESIAN FAULT DISCRIMINATION IN WIDE AREA BACKUP PROTECTION

In many cases, we have certain prior information about the distribution of research objects. Bayesian discriminant analysis is just utilizing this kind of prior information. The basic idea is that all of k categories can be regarded as mutually exclusive subdomains in space, and each observation is a point in this space. Under the precondition of considering prior probability, one can use Bayesian discriminant analysis to construct discriminant functions based on some rules, and calculate the probability of some sample falls into each subdomain respectively. The category with maximum probability is considered to be the class that the sample belongs to.

In our research, the nodal voltage synchronized phasor data in sequence or phase form will be adopted as the input data for the location method. In order to simulate the obtained synchro-phasor data from the PMUs in practice, the software named BPA has been adopted in our paper. The test examples used in the present research, such as IEEE 9 nodes with 3 generators or IEEE 39 nodes with 10

generators, are usually employed and studied in the field of power system and its automation, which are also internationally accepted as the standard power system test cases.

A. Bayesian fault discrimination in IEEE9-Bus system

Fig.1 presents the electric diagram of IEEE 9-bus system.

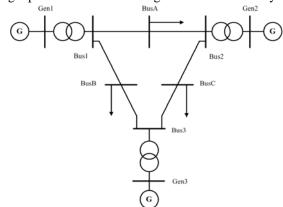


Figure 1 Electric diagram of IEEE 9-bus system

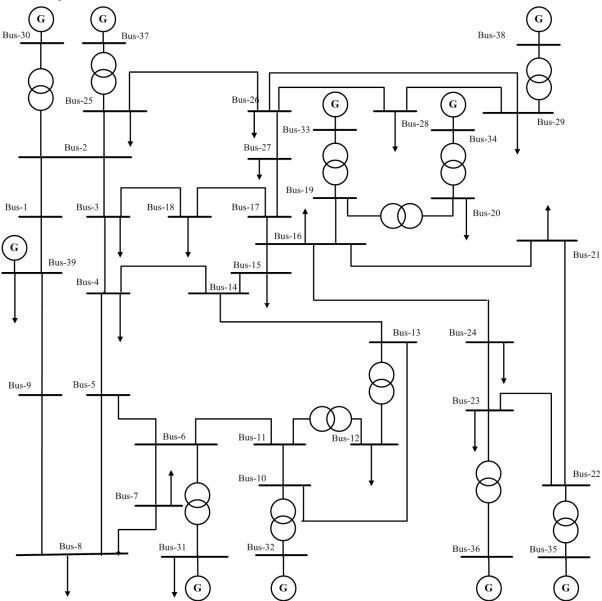


Figure 2 Electric diagram of IEEE 39-bus system

In the structure of electricity grid, Bus1 appears single-phase to ground fault. For IEEE 9-bus system, the node negative sequence voltages at T_{-1} , T_0 (Fault) and T_1 three times can be obtained. According to Bayesian discriminant analysis principles, one calculates the classification function coefficients. Finally the results of posterior probability and discriminant classification based on Bayesian discriminant analysis have been obtained, see Table I.

Similarly, suppose Bus1 appears double-phase to ground fault. According to the node negative sequence voltages, The results of posterior probability and discriminant classification based on Bayesian discriminant analysis have been listed in Table II.

To sum up the above Bayesian discriminant classification results, the misjudgment ratio is,

$$\hat{p}_r^* = \frac{1}{9} = 0.11\dot{1} \tag{10}$$

For single-phase to ground fault and double-phase to ground fault in IEEE 9-bus system, Bayesian discriminant analysis has confirmed fault position. As to misjudged node, in the topology structure BusB is directly connected with preset fault point Bus1. Hence, the branch constructed by the above two nodes could be confirmed as the fault section, which is an acceptable location result. Obviously, if the only fault location needs to be implemented, the node, corresponding to the largest absolute difference of poster probability between fault population and normal population, will be the final result.

Now let us further consider a more complex three-phase grounding fault in IEEE39-Bus system. Fig.2 presents the 10-machine 39-bus New-England Power System. In this system, Bus18 appears three-phase to ground fault. By simulation trials, using these actual measurement data of corresponding variables, the diagnosis of fault component and non-fault component will be completed successfully.

B. Bayesian fault discrimination of three-phase short-circuit fault in IEEE39-Bus system

For IEEE 39-bus system, the node positive sequence voltages at T_{-1} , T_0 (Fault) and T_1 three times can also be obtained. Based on Bayesian discriminant analysis principles, the classification function coefficients have been calculated, and the results of posterior probability and discriminant classification are listed in Table III.

In this simulation example, for three-phase short-circuit fault in IEEE39-Bus system, the misjudgment ratio is,

$$\hat{p}_r^* = \frac{1}{39} = 2.56\% \tag{11}$$

That is to say, the accuracy rate of Bayesian fault discrimination has reached

$$1 - \hat{p}_r^* = 1 - \frac{1}{39} = 97.44\% \tag{12}$$

TABLE I. THE POSTERIOR PROBABILITY AND DISCRIMINANT CLASSIFICATION OF SINGLE-PHASE GROUND FAULT IN IEEE9-BUS SYSTEM

Node	Classification	Posterior probability (Fault population)	Posterior probability (Normal population)	Bayesian discriminant classification
Bus1	F	0.99096	0.00904	F
Bus2	N	0.00275	0.99725	N
Bus3	N	0.00451	0.99549	N
BusA	N	0.41227	0.58773	N
BusB	N	0.55977	0.44023	F
BusC	N	0.00239	0.99761	N
Gen1	N	0.00931	0.99069	N
Gen2	N	0.00029	0.99971	N
Gen3	N	0.00066	0.99934	N

TABLE II. THE POSTERIOR PROBABILITY AND DISCRIMINANT CLASSIFICATION OF DOUBLE-PHASE GROUND FAULT IN IEEE9-BUS SYSTEM

Node	Classification	Posterior probability (Fault population)	Posterior probability (Normal population)	Bayesian discriminant classification
Bus1	F	0.99095	0.00905	F
Bus2	N	0.00274	0.99726	N
Bus3	N	0.00453	0.99547	N
BusA	N	0.41134	0.58866	N
BusB	N	0.5605	0.4395	F
BusC	N	0.0024	0.9976	N
Gen1	N	0.00935	0.99065	N
Gen2	N	0.00029	0.99971	N
Gen3	N	0.00066	0.99934	N

TABLE III. THE POSTERIOR PROBABILITY AND DISCRIMINANT CLASSIFICATION OF THREE-PHASE SHORT-CIRCUIT FAULT IN IEEE39-BUS SYSTEM

Node	Classification	Posterior probability (Fault population)	Posterior probability (Normal population)	Bayesian discriminant classification
Bus-1	N	0	1.0000	N
Bus-2	N	0.00011	0.9999	N
Bus-3	N	0.1568	0.8432	N
Bus-4	N	0.00001	1.0000	N
Bus-5	N	0	1.0000	N
Bus-6	N	0	1.0000	N
Bus-7	N	0	1.0000	N
Bus-8	N	0	1.0000	N
Bus-9	N	0	1.0000	N
Bus-10	N	0	1.0000	N
Bus-11	N	0	1.0000	N
Bus-12	N	0	1.0000	N
Bus-13	N	0	1.0000	N
Bus-14	N	0.00001	0.99999	N
Bus-15	N	0.0001	0.9999	N
Bus-16	N	0.00087	0.99913	N
Bus-17	N	0.90076	0.09924	F
Bus-18	F	1.0000	0	F
Bus-19	N	0	1.0000	N
Bus-20	N	0	1.0000	N
Bus-21	N	0.00003	1.0000	N
Bus-22	N	0	1.0000	N
Bus-23	N	0	1.0000	N
Bus-24	N	0.00042	0.9996	N
Bus-25	N	0.00006	0.9999	N
Bus-26	N	0.0015	0.9985	N
Bus-27	N	0.05882	0.9412	N
Bus-28	N	0.00002	1.0000	N
Bus-29	N	0	1.0000	N
Bus-30	N	0	1.0000	N
Bus-31	N	0	1.0000	N
Bus-32	N	0	1.0000	N
Bus-33	N	0	1.0000	N
Bus-34	N	0	1.0000	N
Bus-35	N	0	1.0000	N
Bus-36	N	0	1.0000	N
Bus-37	N	0	1.0000	N
Bus-38	N	0	1.0000	N
Bus-39	N	0	1.0000	N

Therefore, for node positive sequence voltages of three-phase short-circuit fault in IEEE39-Bus system, Bayesian discriminant analysis has achieved impressively high accuracy. Only one bus is misjudged—"Bus-17". Because it is directly connected with Bus-18, the fault of Bus-18 will undoubtedly affect Bus-17. Actually this misjudgment of Bus-17 can also help us to confirm the fault section.

A large number of simulation examples have confirmed that the results of Bayesian fault discriminant in wide area backup protection are accurate and reliable.

IV. CONCLUSION AND DISCUSSION

Currently, in the novel wide area backup protection, fault location process is playing an important role. The rapid and accurate fault location algorithm could guarantee the enough time left for adjusting the setting value of related protection. With the widely installment of PMU in nowadays power system especially in China, the large amount synchronized phasor data concentrated in the dispatching center need be processed by an efficiently tool to extract the fault features. Multivariate statistical analysis could perform well in satisfying this requirement. Fault location could be regarded as one process to confirm that which group (normal or fault) the each component belongs to. Therefore, the Bayesian discriminant analysis is focused, which can be featured by taking the prior cognition into account. In this paper, based on Bayesian discriminant analysis principle, a new fault location method is proposed. A large number of simulation examples have confirmed that the results of Bayesian fault discriminant in wide area backup protection are accurate and reliable.

REFERENCES

- S. H. Horowitz and A. G. Phadke, "Third zone revisited," IEEE Transactions on Power Delivery, vol.21, pp.23-29, Jan. 2006.
 [Online]. Available: http://dx.doi.org/10.1109/TPWRD.2005.860244
- [2] A. G. Phadke and J. S. Thorp, "Expose hidden failures to prevent cascading outages," IEEE Computer Applications in Power, vol.9, pp.20-23, Jul. 1996. [Online]. Available: http://dx.doi.org/ 10.1109/67.526849
- [3] J.C. Tan, P.A. Crossley, P.G. McLaren, P.F. Gale, I. Hall, J.Farrell, "Application of a wide area backup protection expert system to prevent cascading outages," IEEE Transactions on Power Delivery, vol.17, pp.375-380, Apr. 2002. [Online]. Available: http://dx.doi.org/ 10.1109/61.997902
- [4] A. G. Phadke and R.M. de Moraes, "The wide world of wide-area measurement," IEEE Power & Energy Magazine, vol.6, pp.52-65, Aug. 2008. [Online]. Available: http://dx.doi.org/ 10.1109/MPE.2008.927476
- [5] A. G. Phadke and J. S. Thorp, Synchronized phasor measurements and their applications, Springer Verlag, 2008.
- [6] A. G. Phadke and J. S. Thorp, Computer relaying for power system, Second edition, John Wiley & Sons Ltd, Chichester, 2009.
- [7] K. P. Lien, C.W. Liu, C. S. Y, J. Jiang, "Transmission network fault location observability with minimal PMU placement," IEEE Transactions on Power Delivery, vol.21, pp.1128-1136, Jul. 2006. [Online]. Available: http://dx.doi.org/10.1109/TPWRD.2005.858806
- [8] Z. Q. He, Z. Zhang, W, Chen, O.P. Malik and X. G. Yin, "Wide-Area Backup Protection Algorithm Based on Fault Component Voltage Distribution," IEEE Transactions on Power Delivery, vol.26, pp.2752-2760, Oct. 2011. [Online]. Available: http://dx.doi.org/

- 10.1109/TPWRD.2011.2165971
- [9] M. M. Eissa, M. E. Masoud, M. M.M. Elanwar, "A Novel back up wide area protection technique for power transmission grids using phasor measurement," IEEE Transactions on Power Delivery, vol.25, pp.270-278, Jan. 2010. [Online]. Available: http://dx.doi.org/ 10.1109/TPWRD.2009.2035394
- [10] J. Tang, P.G. Mclaren. "A wide area differential backup protection scheme for shipboard application," IEEE Transactions on Power Delivery, vol.21, pp.1183-1190, Jul. 2005. [Online]. Available: http://dx.doi.org/10.1109/ESTS.2005.1524678
- [11] C. Wang, C. X. Dou, X. B. Li and Q. Q. Jia, "A WAMS/PMU-based fault location technique," Electric Power Systems Research, vol. 77, pp. 936-945, Jun. 2007. [Online]. Available: http://dx.doi.org/10.1016/j.epsr.2006.08.007
- [12] C. Wang, Q. Q. Jia, X. B. Li and C. X. Dou, "Fault location using synchronized sequence measurements," International Journal of Electrical Power & Energy Systems, vol.30. pp. 134-139, Feb.2008. [Online]. Available: http://dx.doi.org/ 10.1016/j.ijepes.2007.06.027
- [13] J. N. Peng, Y. Z. Sun and H. F. Wang, "Optimal PMU placement for full network observability using Tabu search algorithm," International Journal of Electrical Power & Energy Systems, vol. 28, pp. 223-231, May. 2006. [Online]. Available: http://dx.doi.org/ 10.1016/j.ijepes.2005.05.005
- [14] Z. P. Wang, Y. G. Zhang and J. F. Zhang, "Recent research progress in fault analysis of complex electric power systems," Advances in Electrical and Computer Engineering, vol.10, pp.28-33, Feb. 2010. [Online]. Available: http://dx.doi.org/ 10.4316/AECE.2010.01005
- [15] Y. G. Zhang, J. Ma, J. F. Zhang and Z. P. Wang, "Fault diagnosis based on cluster analysis theory in wide area backup protection system," in *Proc. 2009 Asia-Pacific Power and Energy Engineering Conference*, Wuhan, 2009, pp.1-4. [Online]. Available: http://dx.doi.org/10.1109/APPEEC.2009.4918370
- [16] Y. G. Zhang, Z. P. Wang and J. F. Zhang, "Fault discrimination using synchronized sequence measurements under strong white gaussian noise background," International Journal of Emerging Electric Power Systems, vol.12, pp.1-15, Jun. 2011. [Online]. Available: http://dx.doi.org/ 10.2202/1553-779X.2681
- [17] Y. G. Zhang, Z. P. Wang, J. F. Zhang and J. Ma, "Fault localization in electrical power systems: A pattern recognition approach," International Journal of Electric Power & Energy Systems, vol.33, pp.791-798, Mar. 2011. [Online]. Available: http://dx.doi.org/ 10.1016/j.ijepes.2011.01.018
- [18] Y. G. Zhang, Z. P. Wang, J. F. Zhang and J. Ma, "PCA fault feature extraction in complex electric power systems," Advances in Electrical and Computer Engineering, vol.10, pp.102-107, Aug. 2010. [Online]. Available: http://dx.doi.org/10.4316/aece.2010.03017
- [19] Y. Tharrualt, G. Mourot and J. Ragot, "Fault detection and isolation with robust principal component analysis," in *Proc. 16th Mediterranean Conference on Control & Automation*, Ajaccio, 2008, pp. 59-64. [Online]. Available: http://dx.doi.org/ 10.1109/MED.2008.4602224
- [20] Y. G. Zhang, Z. P. Wang and J. F. Zhang, "PCA Fault location based on wide area measurement systems," in *Proc. 5th International Conf. Critical Infrastructure*, Bejing, 2010, pp. 1-4. [Online]. Available: http://dx.doi.org/ 10.1109/CRIS.2010.5617572
- [21] Z. P. Wang, Y. G. Zhang and J. F. Zhang, "Principal components fault location based on WAMS-PMU measure system," in *Proc. 2011 IEEE Power and Energy Society General Meeting*, USA, 2011, pp. 1-5. [Online]. Available: http://dx.doi.org/10.1109/PES.2011.6039102
- [22] J. Cao, Principal component analysis based fault detection and isolation, Ph.D. thesis of George Mason University, 2004.
- [23] J. E. Jackson, A user's guide to principal components, John Wiley and Sons, New York, 1991.
- [24] C. L. Mei and J. C. Fan, Data analysis methods, Higher Education Press, Beijing, 2006.
- [25] Y. G. Zhang, Z. P. Wang and J. F. Zhang, "New type of wide area fault location algorithm based on posterior viewpoints," in *Proc. IASTED International Conference on Power and Energy Systems and Applications*, Pittsburgh, 2011, pp.77-81. [Online]. Available: http://dx.doi.org/ 10.2316/P.2011.756-076