Optimal Power Flow Control by Rotary Power Flow Controller

Amir Mousa HADDADI, Ahad KAZEMI
Electrical Engineering Department, Iran University of Science and Technology
Narmak, Tehran, 16846, Iran
haddadi@elec.iust.ac.ir

***Abstract*—This paper presents a new power flow model for rotary power flow controller (RPFC). RPFC injects a series voltage into the transmission line and provides series compensation and phase shifting simultaneously. Therefore, it is able to control the transmission line impedance and the active power flow through it. An RPFC is composed mainly of two rotary phase shifting transformers (RPST) and two conventional (series and shunt) transformers. Structurally, an RPST consists of two windings (stator and rotor windings). The rotor windings of the two RPSTs are connected in parallel and their stator windings are in series. The injected voltage is proportional to the vector sum of the stator voltages and so its amplitude and angle are affected by the rotor position of the two RPSTs. This paper, describes the steady state operation and single-phase equivalent circuit of the RPFC. Also in this paper, a new power flow model, based on power injection model of flexible ac transmission system (FACTS) controllers, suitable for the power flow analysis is introduced. Proposed model is used to solve optimal power flow (OPF) problem in IEEE standard test systems incorporating RPFC and the optimal settings and location of the RPFC is determined.**

***Index Terms*—flexible ac transmission systems (FACTS), optimal power flow (OPF), power injection model, rotary power flow controller (RPFC), rotary phase shifting transformer (RPST).**

I. INTRODUCTION

Power flow control plays an important role in the steady state operation of an interconnected power system. In its general form, power flows through the network are mainly determined by the voltage amplitudes and phase angles and transmission line impedances [1]. It is obvious that transmission lines with lower impedance take a more share of power flows than those with higher impedance. This case not only is an unpleasant situation, but also results in operational problems, especially under heavily loaded system conditions. System instability, loop flows, high transmission losses, voltage limit violations, inability to utilize transmission line capability up to the thermal limit, cascade tripping and high operational costs has been mentioned as a result of unregulated active and reactive power flows [1]. Upgrading existing transmission lines by using FACTS controllers is suggested as a solution to these problems [2-5].

A variety of FACTS controllers have been introduced in the papers and their steady state models and applications for power flow control well established. Phase shifting transformers (PST), which belong to the first generation of facts devices have been in existence for many years. Steady-state models and applications of PST have been investigated in papers [6, 7]. The second generation of FACTS controllers including Static synchronous series compensator (SSSC) and unified power flow controller (UPFC) are also discussed well [8-11]. These FACTS controllers have definitive abilities but their usage is limited because of the cost considerations. Among these FACTS controllers, PST has the lowest cost and simplest structure; but it has limited abilities in comparison with UPFC. Hybrid flow controller (HFC) is another member of hybrid FACTS controllers. It is composed of a conventional PST, thyristor-switched series capacitors and reactors (TSSC and TSSR) and a mechanically switched shunt capacitor [12].

RPFC is a member of FACTS controllers that is represented as an alternative to the UPFC [13, 14] (mainly because of its lower cost). This paper demonstrates the structure and steady-state operation of the RPFC. It also introduces a new power injection model for the RPFC. RPFC is mainly composed of two RPSTs and two conventional transformers. The utilization of the shunt transformer makes RPFC be able to control reactive power at the bus, which it is installed. Among mentioned FACTS controllers, only UPFC and HFC have this ability. RPFC injects a controllable series voltage into the transmission line like SSSC and UPFC. The amplitude and the phase angle of this voltage both can be controlled by the rotor position of the two RPSTs. Therefore, the response of an RPFC is much faster than a conventional PST that uses mechanical switches and so it can provide dynamic power flow control and improve dynamic stability of the power system. In addition, RPFC provides continuous control of the injected voltage and does not need mechanical switches. This improves its reliability and reduces the repair and maintenance costs. Considering all of these advantages, RPFC is the main competitor of the HFC.

This paper is organized as follows: section II describes the components and the physical structure of the RPFC; section III introduces the steady state operation, single-phase equivalent circuit and vector diagrams of the RPFC and compares its abilities with other FACTS controllers; section IV demonstrates the power injection model of the RPST; section V explains optimal power flow problem formulation, including variables, objective functions and constraints; section VI reports the OPF results for different IEEE standard systems incorporating the RPFC. Optimal settings and best position of the RPFC is also determined in this section.

II. RPFC STRUCTURE

A wound rotor induction machine with p pairs of poles

Digital Object Identifier 10.4316/AECE.2011.02013

can be operated as a phase shifting transformer to produce a voltage with constant frequency at standstill while the rotor windings are left on open circuit. It is interesting to note that, in this condition, the induction machine works as a phase shifting transformer, with the only difference that the voltages induced in the secondary windings are the result of rotating magnetic field produced by the primary windings, and therefore, the magnitude of the secondary voltage is independent of the rotor position; although the relative phasor position of the induced secondary voltages with respect to their primary counterparts are determined by the rotor position. Fig.1 shows a rotary phase shifting transformer (RPST) which the magnetic axes of its primary and secondary windings are displaced from each other by an angle equal to β_{mech}. The phase angles of the secondary voltages are dependent on the electrical position of the rotor with respect to the stator. With many pairs of poles, it is possible to control the phase angle of the secondary voltages continuously and with small displacements in the rotor position. The rotor can be moved 180 electrical degrees.

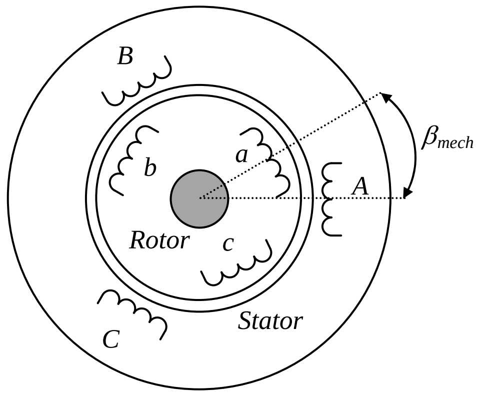

Figure 1. Schematic diagram of the RPST

Fig. 2 shows the main components of the RPFC, which consists of two conventional series and shunt transformers and two RPSTs. We assume that the rotor windings (named primary windings) are fed from the shunt transformer and connected in parallel and the stator windings (named secondary windings) are in series and connected to the series transformer. The phase angle of the stator voltages with respect to the rotor voltages can be adjusted with a simple rotation of the rotor, which is performed by means of a hydraulic motor or a DC motor, coupled on the rotor shaft. The rotor displacement can be made much faster than that of the mechanical switches of a conventional phase shifting transformer and sufficiently fast for steady state and dynamic power flow control.

It is important to note that in an induction machine under normal running conditions, rotor frequency is very low; but when the rotor is blocked (like RPST) rotor frequency is the same as stator frequency; therefore, core loss is increased. In this case, it is better to mount the shaft vertically and since the rotor does not rotate continuously, RPST is sunk in oil for better cooling condition. On the other side, losses due to friction and windage do not exist in the RPST. Also, the design of rotor windings and core may differ from the conventional induction machine. Thus RPST efficiency is improved in comparison with induction machine.

In comparison with PST, the leakage reactance and the magnetizing current of an RPST is more significant due to distributed windings and the air gap between the rotor and the stator, respectively. It is also possible to supply this magnetizing current by shunt capacitors across both stator windings, as shown in Fig. 2. On the other side, RPST is faster than conventional PST and do not need for rotary or slider switches. In addition, RPFC is able to control the voltage continuously in contrast to PST. Therefore, when we need fast, smooth and repetitive power flow control, an RPFC would be better than a conventional phase shifting transformer. RPST is also more suitable for handling fault conditions than PST and SSSC. Omitting the magnetizing current, RPFC does not generate harmonics.

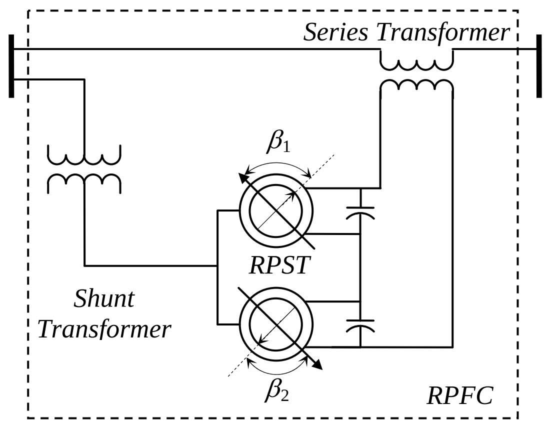

Figure 2. Schematic diagram of the RPFC

III. Steady State Operation of RPFC

RPFC injects a voltage, of variable magnitude and phase angle, in series with the transmission line. This injected voltage can be in quadrature with the line current, thereby imitating an inductive or a capacitive reactance in series with the transmission line. This emulated variable reactance, inserted by injection of series voltage, affects power flow in the transmission line by providing series compensation (like SSSC). Fig. 4.a shows this functional capability of RPFC. Also, the injected voltage can be in quadrature with the bus voltage, thereby causing a phase shift in sending or receiving end voltage. In this way, RPFC acts like a quadrature phase shifter and is able to control power angle. Fig. 4.b displays the vector diagram of an RPFC working as a PST. In addition to stated abilities, RPFC is able to control the bus voltage by injecting a series voltage in-phase with the bus voltage. Fig 4.c shows the capability of terminal voltage regulation of the RPFC. In general, an RPFC is able to control power angle and transmission line impedance simultaneously which none of two mentioned FACTS controller is able to do it. In fact, the control region of an RPFC is comparable to that of a UPFC. Because in addition to series compensation, RPFC is able to provide shunt compensation and control reactive power by the means of shunt branch and shunt capacitors.

Fig. 3 shows single-phase equivalent circuit of the RPFC which is connected between buses *i* and *j* within a transmission line. The rotor windings are connected in parallel and fed from the secondary winding of the shunt transformer. The stator windings and the secondary winding of the series transformer are connected in series.

It is important to note that both phase angle and amplitude of the injected voltage are adjusted by the rotor position of two RPSTs. Accordingly, using tap changing (shunt and series) transformers is optional and only for enhancing the control region of the RPFC.

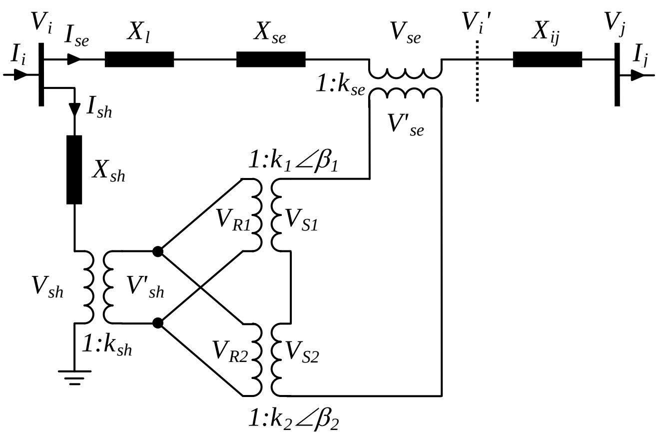

Figure 3. Single-phase equivalent circuit of RPFC

A steady state vector diagram of the RPFC based on Fig. 3 is represented in Fig. 4.d. It is assumed that total reactance in series with the primary winding of the series transformer is equal to X_t; thus $X_t=2X_l+X_{se}+X_{ij}$. V_i and V_j are the voltage phasor of buses i and j. In this diagram, I_{sh} is not shown. RPFC injects an adjustable series voltage V_{se} that is proportional to V'_{se}. V'_{se} is the vector sum of the stator voltages V_{s1} and V_{s2}. On the other side, rotor voltages are equal to V'_{sh}, therefore V_{s1} and V_{s2} are proportional to the secondary voltage of the shunt transformer (V'_{sh}) with a phase shift equal to β_1 and β_2, respectively. Fig. 4 also illustrates the control zone of the RPFC, which is surrounded with the circle having a radius equal to the series injected voltage.

IV. RPFC Power Flow Model

With reference to Fig. 3, the series injected voltage of the RPFC can be expressed as

$$V_{se}=k_{se}V_{se}^{'}=k_{se}\left(V_{S1}+V_{S2}\right) \tag{1}$$

where k_{se} is the series transformer voltage ratio. V_{S1} and V_{S2} are the stator voltage vectors. Since each RPST acts like a phase shifter, (1) can be rewritten as

$$V_{se}=k_{se}\left(\left(k_1\measuredangle\beta_1\right)V_{R1}+\left(k_2\measuredangle\beta_2\right)V_{R2}\right) \tag{2}$$

where k_1 and k_2 are the voltage ratio of the each of the RPSTs; β_1 and β_2 are the amount of rotors displacement with respect to the stator windings in electrical degrees. Supposing an RPST with p pair of poles, relation between rotor movement in electrical degrees and in mechanical degrees comes $\beta=\beta_{elec}=p.\beta_{mech}$.

For the sake of simplicity, the voltage ratios of both RPSTs are assumed the same, which means: $k_1=k_2=k$. So (2) is simplified to

$$V_{se}=k_{se}k\left(e^{j\beta_1}+e^{j\beta_2}\right)V_{sh}^{'} \tag{3}$$

Defining new variables α and γ, the series injected voltage can be expressed as

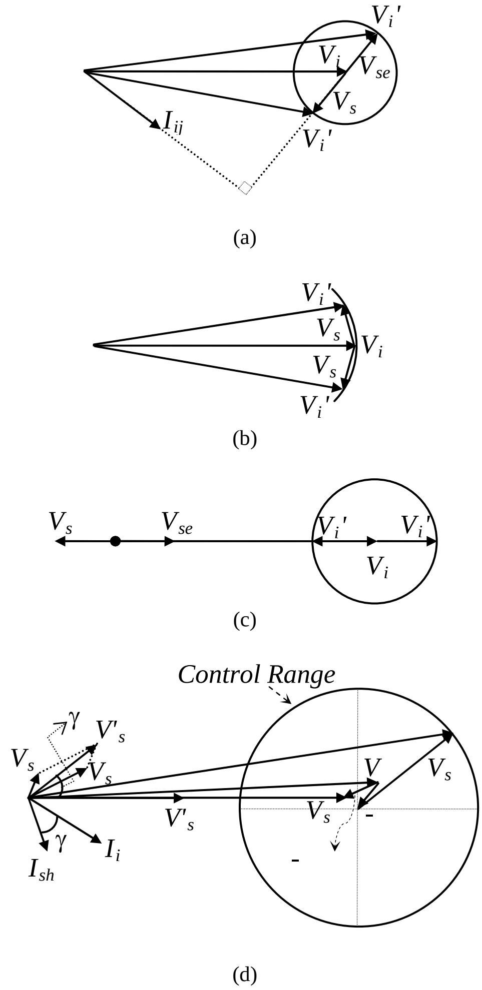

Figure 4. RPFC vector diagram: (a) Series compensation; (b) Phase shifting operation; (c) Bus voltage regulation; (d) General operation and control zone.

$$V_{se}=2k_{se}kCos(\alpha)V_{sh}^{'}e^{j\gamma} \tag{4}$$

with

$$\alpha=\frac{\beta_2-\beta_1}{2} \tag{5}$$

$$\gamma=\frac{\beta_2+\beta_1}{2} \tag{6}$$

Taking the shunt transformer voltage ratio (k_{sh}) into account, we deduce

$$V_{se}=2\frac{k_{se}}{k_{sh}}kCos(\alpha)V_{sh}e^{j\gamma}=k_{eq}e^{j\gamma}V_{sh} \tag{7}$$

It is obvious that k_{eq} is a function of α. With reference to

Fig. 3, the following equation is obtained for the primary voltage of the shunt transformer

$$V_{sh} = V_i - jX_{sh} I_{sh} \tag{8}$$

where X_{sh} is the leakage reactance of the shunt transformer. If we neglect the active and reactive power exchange of the RPFC with the network, which means assuming an ideal RPFC, we can write

$$V_{sh} \times I_{sh}^* = V_{se} \times I_{se}^* \tag{9}$$

From (7) and (9) we deduce

$$I_{sh} = k_{eq} e^{-j\gamma} I_{se} \tag{10}$$

Combining (7) and (8) gives

$$V_{se} = k_{eq} e^{j\gamma} \left(V_i - jX_{sh} I_h \right) \tag{11}$$

Substituting for I_{sh} from (10) in (11)

$$V_{se} = k_{eq} e^{j\gamma} V_i - jX_{sh} k_{eq}^2 I_{se} \tag{12}$$

Applying the Kirchhoff's voltage law in Fig. 3

$$V_j = V_i - j\left(X_{se} + X_{ij} + 2X_l \right) I_{se} + V_{se} \tag{13}$$

where X_l is the leakage reactance of each RPST in p.u. and is referred to the primary of the series transformer. X_{se} is the leakage reactance of the series transformer. X_{ij} is the line reactance between buses *i* and *j*. Substituting for V_{se} from (12) in (13), the relation of I_{se} can be extracted as

$$I_{se} = \frac{\left(1 + k_{eq} e^{j\gamma}\right) V_i - V_j}{j\left(X_{se} + X_{ij} + 2X_l + k_{eq}^2 X_{sh} \right)} \tag{14}$$

Applying the Kirchhoff's current law in Fig. 3

$$I_i = I_{se} + I_{sh} \tag{15}$$

Substituting for I_{se} from (10) in (15) and then Substituting for I_{se} , we obtain

$$I_i = \frac{\left(1 + 2k_{eq} Cos(\gamma) + k_{eq}^2\right) V_i - \left(1 + k_{eq} e^{-j\gamma}\right) V_j}{jX_{tot}} \tag{16}$$

where $X_{tot}=X_{se}+X_{ij}+2X_l+k_{eq}{}^2X_{sh}$. In order to keep original structure and symmetry of the admittance matrix, with respect to Fig. 5 that shows a general power injection model for FACTS controllers, we express I_i as follows

$$I_i = I_{ij} + I_{ii} \tag{17}$$

with the following equations for I_{ii} and I_{ij}

$$I_{ij} = \left(\frac{V_i - V_j}{jX_{ij}} \right) \tag{18}$$

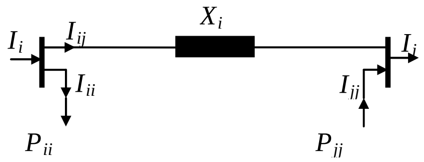

Figure 5. Power injection model of FACTS

$$I_{ii} = \left(\frac{1 + 2k_{eq} Cos(\gamma) + k_{eq}^2}{jX_{tot}} - \frac{1}{jX_{ij}} \right) V_i + \left(\frac{1}{jX_{ij}} - \frac{1 + k_{eq} e^{-j\gamma}}{jX_{tot}} \right) V_j . \tag{19}$$

It is very easy to take into account the line charging susceptance with this model. The complex power at the bus *i* can be stated as

$$S_i = V_i I_i^* \tag{20}$$

and

$$S_{ii} = V_i I_{ii}^* = j\left(\frac{1 + 2k_{eq} Cos(\gamma) + k_{eq}^2}{X_{tot}} - \frac{1}{X_{ij}} \right) \left|V_i\right|^2 + j\left(\frac{1}{X_{ij}} - \frac{1 + k_{eq} e^{j\gamma}}{X_{tot}} \right) V_i V_j^* . \tag{21}$$

If the voltage phasor of buses *i* and *j* are supposed to be $V_i\angle\theta_i$ and $V_j\angle\theta_j$, active and reactive powers at bus *i* can be obtained as follows

$$P_{ii} = \frac{k_{eq} Sin\gamma}{X_{tot}} \left|V_i\right| \left|V_j\right| Cos\theta_{ij} - \left(\frac{1}{X_{ij}} - \frac{1 + k_{eq} Cos\gamma}{X_{tot}} \right) \left|V_i\right| \left|V_j\right| Sin\theta_{ij} \tag{22}$$

$$Q_{ii} = \left(\frac{1 + 2k_{eq} Cos(\gamma) + k_{eq}^2}{X_{tot}} - \frac{1}{X_{ij}} \right) \left|V_i\right|^2 + \frac{k_{eq} Sin\gamma}{X_{tot}} \left|V_i\right| \left|V_j\right| Sin\theta_{ij} - \left(\frac{1}{X_{ij}} - \frac{1 + k_{eq} Cos\gamma}{X_{tot}} \right) \left|V_i\right| \left|V_j\right| Cos\theta_{ij} \tag{23}$$

with $\theta_{ij}=\theta_i-\theta_j$.With reference to Fig. 3, we have

$$I_j = I_{se} = \frac{\left(1+k_{eq}e^{j\gamma}\right)V_i - V_j}{jX_{tot}} \tag{24}$$

In a similar way explained before, with respect to Fig. 5 we can write

$$I_{jj} = \frac{\left(1+k_{eq}e^{j\gamma}\right)V_i - V_j}{jX_{tot}} - \frac{V_i}{jX_{ij}} + \frac{V_j}{jX_{ij}} \tag{25}$$

or

$$I_{jj} = \left(\frac{1+k_{eq}e^{j\gamma}}{jX_{tot}} - \frac{1}{jX_{ij}}\right)V_i + \left(\frac{1}{jX_{ij}} - \frac{1}{jX_{tot}}\right)V_j \tag{26}$$

and

$$S_{jj} = j\left(\frac{1+k_{eq}e^{-j\gamma}}{X_{tot}} - \frac{1}{X_{ij}}\right)V_j V_i^* + j\left(\frac{1}{X_{tot}} - \frac{1}{X_{ij}}\right)V_j^2 \tag{27}$$

Finally we obtain

$$P_{jj} = \frac{k_{eq}Sin\gamma}{X_{tot}}\left|V_i\right|\left|V_j\right|Cos\theta_{ij} - \left(\frac{1}{X_{ij}} - \frac{1+k_{eq}Cos\gamma}{X_{tot}}\right)\left|V_i\right|\left|V_j\right|Sin\theta_{ij} \tag{28}$$

and

$$Q_{jj} = \left(\frac{1}{X_{ij}} - \frac{1}{X_{tot}}\right)\left|V_j\right|^2 - \left(\frac{1}{X_{ij}} - \frac{1+k_{eq}Cos\gamma}{X_{tot}}\right)\left|V_i\right|\left|V_j\right|Cos\theta_{ij} - \frac{k_{eq}Sin\gamma}{X_{tot}}\left|V_i\right|\left|V_j\right|Sin\theta_{ij} \tag{29}$$

Comparing equations (20) and (26), it can be seen that the net active power injection of the RPFC is zero and this is what we expected because RPFC does not generate active power and on the other side, the active power loss of the RPFC is neglected.

β_1 and β_2 are the two main independent control variables of RPFC. If tap changing transformers are utilized, the voltage ratio of these transformers k_{se} and k_{sh} can be also considered as control variables. It is important to note that k_{se} and k_{sh} change in discrete steps, but β_1 and β_2 change continuously. Usually discrete variables cause problems for OPF algorithm convergence. Since the amplitude and phase angle of the injected voltage is fully controlled by the rotor position of the two RPSTs, using tap changing transformer is not necessary. So k_{sh} and k_{se} are assumed constant.

V. Optimal Power Flow Formulation

One of the most important problems in the field of power system operation is to determine the appropriate operating strategies. Optimal power flow is a very useful tool to meet these important practical strategies. OPF can be expressed in the terms of conventional power flow in addition to optimizing an objective function. Therefore, voltage limits, active and reactive power generation limits, network equations, loading conditions, line flow limits, physical limits of FACTS controllers (if embedded) are all included in an OPF problem. Selection of the objective function depends on the aim of operator. It is common to choose active power generation cost as the objective function to be minimized, because economic aspects are very important in power systems. OPF as stated above stands for a static, nonlinear optimization problem. If we suppose the power system with embedded FACTS controllers, which may have discrete control parameters, OPF will be expressed as a mixed integer nonlinear programming.

In its general form, OPF problem can be expressed as the optimization of the objective function $f(x,y)$, Subject to equality constraint $g(x,y)=0$ and inequality constraint $h(x,y)\geq 0$, where x and y are the vectors of independent and dependent variables and $g(x,y)$ denotes the power flow equations and $h(x,y)$ stands for state variable limits and physical constraints.

A. Variables

In an OPF problem, we encounter with two groups of variables: control variables or independent variables and dependent variables. Voltage phasor on the slack bus, active power generation and voltage amplitude on each PV (generator) bus, tap position of tap changing transformers and control parameter of FACTS controllers can be mentioned as control variables. On the other side, Voltage phasor on each PQ (load) bus and voltage angle on each PV bus can be mentioned as dependent variables. Active and reactive powers on each PQ (load) bus are assumed constant parameters, since we are analyzing static OPF.

B. Objective Function

The main goal of the OPF solution is to find the values of control variables that optimize an objective function. Therefore, selection of the objective function is very important. The most popular objective function is the active power generation cost. If we suppose the thermal generation unit costs to be a quadratic polynomial, the objective function to be minimized can be stated as

$$F_T = \sum_{i=1}^{NG} F_i(P_{gi}) = \sum_{i=1}^{NG} a_i + b_i P_{gi} + c_i P_{gi}^2 \tag{30}$$

with

NG Number of generators

a_i , b_i , c_i Cost constant coefficients of unit i

P_i Active power generated by unit i

It is important to include the power generated by the slack bus in (28).

Another objective function is active power losses. Considering this objective function is useful to show the capability of the RPFC to reduce losses. In this case the difference between total active power generation and demand is minimized.

Average system loadability is chosen as another objective function to be minimized. It can be defined as [15]

$$f = \frac{1}{NL}\sum_{i=1}^{NL}\frac{S_i}{S_{i\max}} \quad (31)$$

with

NL Total number of lines

S_i Apparent power flow on line i

S_{imax} maximum apparent power flow on line i

If the apparent power flow on the line i from the sending end to the receiving end and vice versa are named S_{isr} and S_{irs}, the objective function can be defined as [15]

$$f = \frac{1}{2NL}\sum_{i=1}^{NL}\frac{S_{isr}+S_{irs}}{S_{i\max}} \quad (32)$$

C. Equality Constraints

OPF solution must satisfy the power flow equations; unless the OPF results are invalid and the problem is infeasible. The equality constraints include bus real power balance and bus reactive power balance and can be stated as

$$P_{gi} - P_{di} + P_{Fi} = P_i(x,y) \quad , \quad i = 1,2,...,NB \quad (33)$$

$$Q_{gi} - Q_{di} + Q_{Fi} = Q_i(x,y) \quad , \quad i = 1,2,...,NB \quad (34)$$

with

NB Number of buses

P_i Active power injection at bus i

Q_i Reactive power injection at bus i

P_{gi} Active power generation at bus i

Q_{gi} Reactive power generation at bus i

P_{di} Active power consumption at bus i

Q_{di} Reactive power consumption at bus i

P_{Fi} Active power injection of FACTS controller at bus i

Q_{Fi} Reactive power injection of FACTS controller at bus j

As you can see, the effect of FACTS controller is considered in these equations. It is obvious that P_{Fi} and Q_{Fi} are zero for all buses except those where a FACTS controller is installed.

D. Inequality constraints

The inequality constraints relate to upper and lower limit of each variable, which must be satisfied. These constraints can be classified into several categories: system security constraints, i.e. transmission lines loading, bus voltage constraints, the operational constraints of the FACTS controllers, generator security constrains, i.e. real and reactive power outputs. These constraints can be described mathematically as

$$V_i^{\min} \le V_i \le V_i^{\max} \quad , \quad i = 1,2,...,NB \quad (35)$$

$$P_{gi}^{\min} \le P_{gi} \le P_{gi}^{\max} \quad , \quad i = 1,2,...,NG \quad (36)$$

$$Q_{gi}^{\min} \le Q_{gi} \le Q_{gi}^{\max} \quad , \quad i = 1,2,...,NG \quad (37)$$

$$S_i \le S_{i\max} \quad , \quad i = 1,2,...,NL \quad (38)$$

VI. CASE STUDIES

The optimal power flow problem is implemented in GAMS and non-linear programming (NLP) method is employed to solve it. Three different objective functions are considered (total fuel cost, active power loss and average loadability) to be minimized. In each case optimal settings of the RPFC and its best location are determined. RPFC performance is tested on the IEEE 30-bus and 118-bus systems. The results are prepared as follows:

A. IEEE 30-bus system

IEEE 30-bus test system [16] is used to show the capability of the RPFC to control power flow and improve operating condition of the power system while power flow constraints are satisfied. RPFC parameters and limits are given in Appendix. Total active and reactive power demands are 283.4 MW and 126.2 MVAr respectively.

The RPFC optimal settings and its best location for different objective functions are shown in Table I. The rotor angles (β_1 and β_2) are in electrical degrees. For an RPST with p pairs of poles, rotor displacement is obtained from $\beta_{mech} = \beta / p$. The difference between these two angles ($\beta_{21} = \beta_2 - \beta_1$) affects the amplitude of the series injected voltage. With reference to (7) the amplitude of the series injected voltage is a function of $Cos(0.5\beta_{21})$. As β_{21} increases, the amplitude of the series injected voltage decreases. Thus, the series injected voltage is maximized when cost is minimized.

TABLE I. RPFC OPTIMAL SETTINGS AND LOCATION

Objective function	β_1 ($^{\circ e}$)	β_2 ($^{\circ e}$)	Line
Total fuel cost	78.94	118.16	2-5
Active power losses	143.4	33.57	2-5
Average loadability	149.4	44.43	2-5

Tables II, III, IV show the OPF results before and after installing an RPFC. In these tables active and reactive power losses (P_{Loss} and Q_{Loss}) are calculated by I^2Z. Line flow, sending end voltage and receiving end voltage of the line, which RPFC is installed, are also declared. RPFC power rating and series and shunt voltages are shown in this table. Maximum series injected voltage of the RPFC is assumed 0.4 pu.

If the total fuel cost is chosen as the objective function, results show that utilizing an RPFC causes total cost, active power losses and average loadability to reduce. While the goal is to minimize active power losses, the cost is increased that is not suitable.

TABLE II. OPF RESULTS BEFORE AND AFTER INSTALLING RPFC

Total fuel cost minimization		Without RPFC	With RPFC
Min. Cost ($/h)		802.22	790.77
P_{Loss} (MW)		9.44	6.35
Q_{Loss} (MVAr)		37.69	64.06
Average loadability		0.347	0.336
Total P_G (MW)		292.8	289.7
Total Q_G (MW)		103.8	129.7
Bus Voltages (pu)	Bus 2	1.042∠12.251	1.040∠10.676
	Bus 5	1.015∠5.544	1.043∠10.125
Line 2-5 flow (MVA)		63.02	104.91
RPFC rating (MVA)		-	110.8
Series voltage (V_{se})		-	0.369 pu
Shunt voltage (V_{sh})		-	1.033 pu

TABLE III. OPF RESULTS BEFORE AND AFTER INSTALLING RPFC

Active power loss minimization		Without RPFC	With RPFC
Cost ($/h)		968.1	965.09
Min. P_{Loss} (MW)		3.28	2.02
Q_{Loss} (MVAr)		17.07	26.32
Average loadability		0.306	0.294
Total P_G (MW)		286.7	285.4
Total Q_G (MW)		81.8	91.4
Bus Voltages (pu)	Bus 2	1.056∠13.26	1.045∠7.771
	Bus 5	1.036∠9.226	1.045∠7.525
Line 2-5 flow (MVA)		39.4	65.3
RPFC rating (MVA)		-	41.7
Series voltage (V_{se})		-	0.227 pu
Shunt voltage (V_{sh})		-	1.042 pu

TABLE IV. OPF RESULTS BEFORE AND AFTER INSTALLING RPFC

Average loadability minimization		Without RPFC	With RPFC
Cost ($/h)		920.27	917.48
P_{Loss} (MW)		4.35	2.75
Q_{Loss} (MVAr)		21.65	31.66
Min. average loadability		0.285	0.278
Total P_G (MW)		287.7	286.2
Total Q_G (MW)		88.3	98.3
Bus Voltages (pu)	Bus 2	1.045∠6.051	1.036∠9.391
	Bus 5	1.006∠1.789	1.034∠8.844
Line 2-5 flow (MVA)		43.35	67.53
RPFC rating (MVA)		-	46.8
Series voltage (V_{se})		-	0.238 pu
Shunt voltage (V_{sh})		-	1.033 pu

Fig. 6 shows the voltage profiles with and without RPFC while minimizing total fuel cost. According to Table II, reactive power generation is increased by using an RPFC. As a result, the voltage profile is moved up.

B. IEEE 118-bus system

RPFC is used to control power flow in the IEEE 118-bus system [16]. Line ratings are taken from [17]. RPFC parameters are the same as used for the IEEE 30-bus system except that the shunt transformer voltage ratio is equal to 5.52 (138/25 kV). If the RPFC is installed in a line connecting two 345 kV buses, series and shunt transformer voltage ratios are supposed to be 2.5 (125/50 kV) and 13.8 (345/25 kV) respectively.

Total active and reactive power demands are 4242 MW and 1438 MVAr respectively. Table V shows the optimal location and settings of the RPFC. Results show that the rotor angles of the two RPSTs are the same. This means that RPFC reached its limit. So it is possible to reduce cost more by utilizing an RPFC with higher ratings or using more than one RPFC.

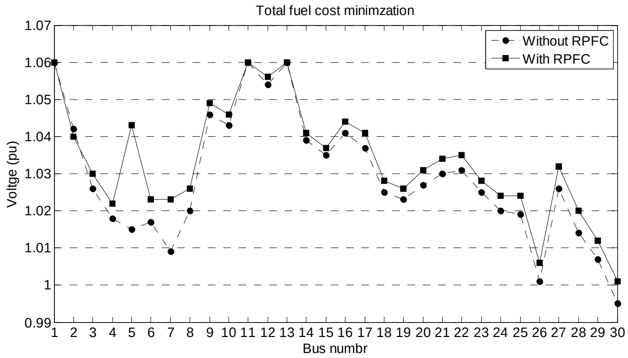

Figure 6. Voltage profile of IEEE 30-bus system

TABLE V. RPFC OPTIMAL SETTINGS AND LOCATION

Objective function	β_1 (°^{e})	β_2 (°^{e})	Line
Total fuel cost	80.45	80.45	25-27
Active power losses	75.74	75.74	80-96

Tables VI and VII show OPF results before and after installing an RPFC.

TABLE VI. OPF RESULTS BEFORE AND AFTER INSTALLING RPFC

Total fuel cost minimization		Without RPFC	With RPFC
Min. Cost ($/h)		129660.69	129392.6
P_{Loss} (MW)		77.4	71.51
Q_{Loss} (MVAr)		483.52	541.19
Average loadability		0.159	0.159
Total P_G (MW)		4319.4	4313.5
Total Q_G (MW)		388.3	441
Bus Voltages (pu)	Bus 25	1.06	1.060
	Bus 27	1.041	1.060
Line 25-27 flow (MVA)		125.44	181.53
RPFC rating (MVA)		-	178.7
Series voltage (V_{se})		-	0.379 pu
Shunt voltage (V_{sh})		-	1.049 pu

TABLE VII. OPF RESULTS BEFORE AND AFTER INSTALLING RPFC

Active power loss minimization		Without RPFC	With RPFC
Cost ($/h)		166388.61	166510.48
Min. P_{Loss} (MW)		9.23	7.85
Q_{Loss} (MVAr)		46.22	83.48
Average loadability		0.074	0.073
Total P_G (MW)		4251.2	4249.8
Total Q_G (MW)		-55.3	-19.5
Bus Voltages (pu)	Bus 80	1.06	1.060
	Bus 96	1.04	1.057
Line 80-96 flow (MVA)		24.43	129.29
RPFC rating (MVA)		-	123.3
Series voltage (V_{se})		-	0.381 pu
Shunt voltage (V_{sh})		-	1.052 pu

If the maximum series injected voltage is increased to 0.7 pu (k_{se}=1.93), OPF results show that rotor angles are 92.63 and 72.92 electrical degrees. In this case minimum cost is decreased to 129328.36 $/h, but power rating of the RPFC is increased to about 500 MVA.

Fig. 7 shows the value of the objective function, which is total fuel cost, versus iteration number for IEEE 118-bus system. Starting from an infeasible solution, after 11 iterations a feasible solution is found. The total number of iterations to find the optimal solution is 44.

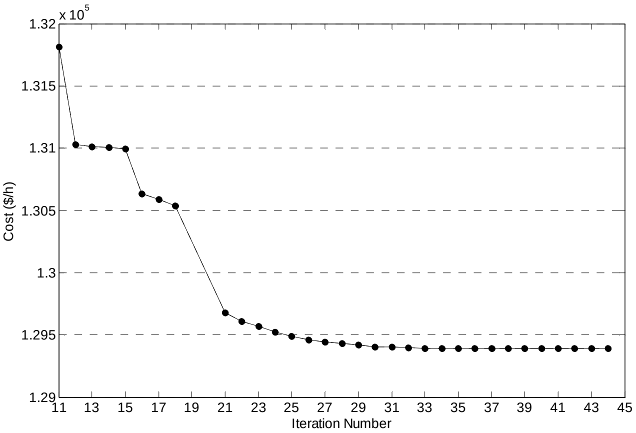

Figure 7. Objective function value versus iteration number

VII. Conclusion

Power flow control through the network is an essential operational requirement. RPFC is a member of FACTS controllers that provides power flow control with some advantages like continuous control, fast response, and widespread control range. This paper presents a new power flow model for an RPFC. The proposed model is based on the power injection model of FACTS controllers. Steady state operation and single-phase equivalent circuit of the RPFC are also developed and a comparison between RPFC and PST is provided. Rotor angles with respect to the stator windings are the two control variables of the RPFC. These two angles control the RPFC operation and are changed by a prime mover.

The OPF model is implemented in GAMS by using the proposed model. NLP method is used to solve OPF problem in IEEE 30-bus and 118-bus systems incorporating the RPFC. Three different objective functions are considered to be minimized: total fuel cost, active power loss and average loadability. Alongside determining the optimal settings of the RPFC which are rotor angles, its best position is obtained.

Simulation results show that RPFC is able to control power flow effectively and minimize the desired objective function. An RPFC with the rated power from less than 110 MVA is enough for IEEE 30-bus system. But IEEE 118-bus test system needs more than three RPFCs with the same power rating or an RPFC with the power rating about 500 MVA. In this study the voltage ratio of the series transformer is assumed 1. Thus it seems that in this case we can eliminate series transformer and simplify the RPFC.

Appendix

Characteristics of the RPFC

RPFC characteristics are chosen based on [14] as follows:

X_{se}: Leakage reactance of the series transformer equal to 0.02 p.u.

k_{se}: Voltage ratio of the series transformer equal to 1 (50/50 kV).

X_{sh}: Leakage reactance of the shunt transformer equal to 0.02 p.u.

k_{sh}: Voltage ratio of the shunt transformer equal to 5.28 (132/25 kV).

X_l: Leakage reactance of each RPST equal to 0.08 p.u.

k: Voltage ratio of each RPST equal to 1 (25/25 kV).

References

[1] E. Acha, C. R. Fuerte-Esquivel, H. Ambriz-Perez, and C. Angeles-Camacho, "FACTS, Modeling and Simulation in Power Networks," John Wiley & Sons Ltd, England, Feb. 2004, pp. 1-28.

[2] N. Li, Y. Xu, and H. Chcn, "FACTS-Based Power Flow Control in Interconnected Power Systems," IEEE Trans. Power Sys., Vol. 15, No. 1, Feb. 2000, pp. 257-262.

[3] S. Y. Ge, and T. **S.** Chung, "Optimal Active Power Flow Incorporating Power Flow Control Needs In Flexible Ac Transmission Systems," IEEE Trans. Power Sys., Vol. 14, No. 2, May 1999, pp. 738-744.

[4] D. J. Gotham, and G. T. Heydt, "Power Flow Control and Power Flow Studies for Systems with FACTS devices," IEEE Trans. Power Sys., Vol. 13, No. 1, Feb. 1998, pp. 60-65.

[5] Y. Xiao, Y. H. Song, and Y. Z. Sun, "Power Flow Control Approach to Power Systems With Embedded FACTS Devices," IEEE Trans. Power Sys., Vol. 17, No. 4, Nov. 2002, pp. 943-950.

[6] M. Noroozian, and G. Andersson, "Power Flow Control by Use of Controllable Series Components," IEEE Trans. Power Del., Vol. 8, No. 3, Jul. 1993, pp. 1420-1429.

[7] M. R. Iravani, Paul L. Dandeno, D. Maratukulam, K.H. Nguyen, and D. Zhu, “Applications of Static Phase Shifters in Power Systems”, IEEE Trans. Power Del., Vol. 9, No. 3, Jul. 1994, pp. 1600-1608.

[8] L. Gyugyi, T. R. Rietman, A. Edris, C. D. Schauda, D. R. Torgerson, and S. L. Williams, "The Unified Power Flow Controller: a New Approach to power Flow Control," IEEE Trans. Power Del., Vol. 10, No. 2, Apr. 1995, pp. 1085-1097.

[9] M. Noroozian, L. Angquist, M. Ghandhari, and G. Andersson, "Use Of UPFC for Optimal Power Flow Control," IEEE Trans. Power Del., Vol. 12, No. 4, Oct. 1997, pp. 1629-1634.

[10] X. P. Zhang, "Advanced Modeling of the Multi-control Functional Static Synchronous Series Compensator (SSSC) in Newton Power Flow," IEEE Trans. Power Sys., Vol. 18, No. 4, Nov. 2003, pp. 1410-1416.

[11] K. K. Sen, and E. J. Stacey, "UPFC - Unified Power Flow Controller: Theory, Modeling, and Applications," IEEE Trans. Power Delivery, Vol. 13, No. 4, Oct. 1998, pp. 1453-1460.

[12] R. Iravani, and M. Noroozian, "Power-Flow Model and Steady-State Analysis of the Hybrid Flow Controller," IEEE Trans. Power Del., Vol. 23, No. 4, Oct. 2008, pp. 2330-2338.

[13] A. O. Ba, T. Peng, and S. Lefebvre, "Rotary Power-Flow Controller for Dynamic Performance Evaluation-Part I: RPFC Modeling," IEEE Trans. Power Del., Vol. 24, No. 3, Jul. 2009, pp. 1406-1416.

[14] A. O. Ba, T. Peng, and S. Lefebvre, "Rotary Power-Flow Controller for Dynamic Performance Evaluation-Part II: RPFC Application in a Transmission Corridor," IEEE Trans. Power Del., Vol. 24, No. 3, Jul. 2009, pp. 1417-1425.

[15] W. Shao, and V. Vittal, "LP-Based OPF for Corrective FACTS Control to Relieve Overloads and Voltage Violations," IEEE Trans. Power Sys., Vol. 21, No. 4, Nov. 2006, pp. 1832-1839.

[16] Power system test case archive, University of Washington, Available: http://www.ee.washington.edu/research/pstca. Available online: http://motor.ece.iit.edu/data