Orthogonal Functions Applied in Antenna Positioning

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***Abstract*—In this paper, we present a method for designing orthogonal, Legendre type filters. Realization of these filters is very simple and they are very fast, robust and precise. They can be used for generating the sequence of Legendre orthogonal functions. We have also developed a new method for positioning an antenna system, based on these filters, where the filter is applied in detection of electromagnetic field gradient. Control algorithm is based on improved method of gradients. Proposed control algorithm has been verified on practically realized, experimental antenna system and compared with some others tracking control algorithms. Performed experiments have verified efficiency, speed and accuracy of the proposed control method.**

***Index Terms*—Antenna System, Gradient Method, Orthogonal Function, Orthogonal Filter.**

I. INTRODUCTION

Since the origins of antenna systems, used for receiving the signal from moving transmitters, control methods for antenna positioning have been developed. Transmitters can be various: ground (moving vehicles), air (airplanes, rockets), or satellites. Basic goal of these control systems is to turn the antenna in direction of transmitter, i.e., in direction of the strongest electromagnetic field. In order to realize this goal, different methods can be used: extremal, optimal, or adaptive control [1]-[3]. The most complex control of antenna systems [4] appears in the case of high-speed moving transmitters with nonlinear trajectory. In the recent time, many authors suggested intelligent control methods.

No matter what control method is chosen, we need to resolve two basic problems: detection of the electromagnetic field gradient and target tracking based on certain algorithm [5]. In practice, method of synchronous detection based on the sequence of trigonometric functions is often used for gradient determining. During gradient detection using trigonometric orthogonal functions (most commonly used for mobile antenna positioning) some disturbances may occur. In fact, it is known that antenna receivers work on the principle of resonance, caused by the signals detected in receiver circuits. Orthogonal trigonometric signals generated for gradient detection can cause unwanted and destructive resonances in receiver circuits, leading to malfunctioning in control systems for antenna positioning.

In this paper, we present a new method for gradient detection based on Legendre type orthogonal filters. Detection is continuous and very fast, due to filter realization in analog technique. Realization of target tracking algorithm is based on improved gradient method. Because of realization method, high operating speed and robustness of developed system, it is very suitable for embedded control systems in rockets, air navigation, and extremal control of rapid technological processes.

This paper is organized as follows. In Section 2 we shortly describe method for obtaining classical orthogonal rational functions by using a new transformation. Section 3 presents orthogonal, Legendre type filters, based on orthogonal rational functions. In Section 4 we propose a new control algorithm for positioning antenna system with practical realization and experimental verification given in Section 5.

II. ORTHOGONAL RATIONAL FUNCTIONS

The history of orthogonal polynomials is very old [6], [7]. Legendre polynomials and their orthogonal properties were established during eighteenth century. The applications of the classical orthogonal polynomials in technical fields as electrical network synthesis, electronics, telecommunication, signal processing theory, control system theory, and process identification are well known [8], [9]. Laplace transforms of the classical polynomials or their modifications are rational functions, which can be easily factorized. This property is very convenient in designing simple procedures for constructing signal generators, orthogonal filters or adaptive controllers.

First, consider a general approach to designing orthogonal functions and orthogonal filters. We consider the rational function $W_n(s)=\prod_{i=1}^{n-1}(s-z_i)/\prod_{i=1}^{n}(s-s_i)$. Let area (domain) D_p inside the complex plain *s,* bounded by contour C_p, contain all the poles of rational function $W_n(s)$. Similarly, area D_z, bounded by contour C_z, contains all the zeros of rational function. The general analysis and results in the field of orthogonal rational functions are given in [10], [11]. Notice that all orthogonal rational functions have zeroes and poles in strictly defined correlations. Zeroes of the rational functions can be obtained by defined transformation of the poles using the determined relation $F(s,\bar{s})=0$, i.e., $\bar{s}=f(s)$. The rational functions sequences are orthogonal if the condition of symmetry of the above relation is fulfilled [10], [11]. Hence, for the rational functions, now we have:

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$$W_n(s) = \frac{\prod_{i=1}^{n-1}\left(s - f(s_i)\right)}{\prod_{i=1}^{n}(s - s_i)} \quad (1)$$

where s_i represent poles and $f(s_i)$ corresponding zeroes of the function $W_n(s)$. By using transformation $f(s)$, poles from domain D_p are being transformed into the zeroes located inside the domain D_z (Figure 1). Thereby, the necessary condition is: $D_p \cap D_z = \varnothing$. By using transform $\bar{s} = f(s)$ and property of symmetry, according to (1):

$$W_n(\bar{s}) = \bar{W}_n(s) = \frac{\prod_{i=1}^{n-1}(s - s_i)}{\prod_{i=1}^{n}\left(s - f(s_i)\right)} \quad (2)$$

Note that $\bar{W}_n(s)$ has poles equal to zeros of $W_n(s)$ and vice versa.

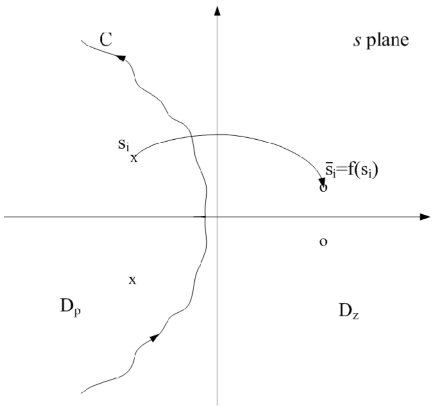

Figure 1. Mapping poles into zeroes using transformation *f*(*s*).

Now, let's prove that the sequence of rational functions is orthogonal inside the complex plane *s*. Consider inner product:

$$\oint_{C_p} W_n(s)\bar{W}_m(s)\,ds = \oint_{C_p} \frac{\prod_{i=1}^{n-1}\left(s - f(s_i)\right)}{\prod_{i=1}^{n}(s - s_i)} \frac{\prod_{i=1}^{m-1}(s - s_i)}{\prod_{i=1}^{m}\left(s - f(s_i)\right)}\,ds \quad (3)$$

By applying the Cauchy theorem, we can obtain the following relation:

$$\oint_{C_p} W_n(s)\bar{W}_m(s)\,ds = \begin{cases} 0, & \text{for } m \neq n \\ N_n \neq 0, & \text{for } m = n \end{cases} \quad (4)$$

hence for $m \neq n$, product of $W_n(s)$ and $\bar{W}_m(s)$ contains no zeroes inside domain D_p. In the case $m \neq n$, all poles in domain D_p (inside the contour C_p) are being eliminated in (4), so according to Cauchy theorem: $\oint_{C_p} W_n(s)\bar{W}_m(s)\,ds = 0$. For $m = n$ all the poles are eliminated except one: $\oint_{C_p} W_n(s)\bar{W}_m(s)\,ds = N_n \neq 0$, where $N_n = 2\pi i \operatorname{Res}_{s=s_n}\left(W_n(s)\bar{W}_m(s)\,ds\right)$.

Function $F(s,\bar{s})$ transforms the area inside the complex plane *s*, wherein the poles are located, into the different area, which contains the poles. Two basic cases of this transformation are: $\bar{s} + s = 0$, i.e., $\bar{s} = -s$, and $\bar{s}s = 1$, i.e., $\bar{s} = \frac{1}{s}$. Transformations that are more general are also possible, whereby the only condition is for the function $F(s,\bar{s})$ to be symmetrical. For instance, transformation $s + \bar{s} = k$, gives all classical orthogonal functions: Legendre, Chebyshev, and Laguerre.

III. ORTHOGONAL FILTERS

Theory of classical orthogonal filters derived from classical orthogonal polynomials (Legendre, Chebyshev, Laguerre…) is well described in many publications [11]-[14]. For designing these filters [12], [15]-[17] shifted Legendre polynomials can be used, which are orthogonal over interval (0, 1). On the other side, technical systems, which are designed using orthogonal polynomials, operate in the real time, so we need the corresponding approximation over interval (0, ∞). For example, by substituting $x = e^{-t}$ into polynomials orthogonal over (0, 1), we obtain the exponential polynomials orthogonal over interval (0, ∞). Then, after applying the Laplace transform to the exponential orthogonal polynomials, orthogonal rational functions can be obtained.

In our case, consider the orthogonal, shifted, Legendre polynomials orthogonal over interval (0, 1), in their explicit form [16]-[18]:

$$P_n(x) = \frac{1}{n!}\sum_{k=0}^{n}(-1)^{n-k}\binom{n}{k}\frac{(n+k)!}{k!}x^k \quad (5)$$

So, the first few members of the Legendre type orthogonal polynomials sequence are the following:

$$P_0(x) = 1,$$
$$P_1(x) = 2x - 1,$$
$$P_2(x) = 6x^2 - 6x + 1,$$
$$P_3(x) = 20x^3 - 30x^2 + 12x - 1, \ldots$$

After applying the substitution $x = e^{-t}$ into (5) and Laplace transform, the following rational function can be obtained:

$$\mathcal{L}\left[P_n\left(e^{-t}\right)\right] = W_n(s) \quad (6)$$

where:

$$W_n(s) = \frac{1}{s}\frac{\prod_{i=1}^{n}(s - i)}{\prod_{i=1}^{n}(s + i)} \quad (7)$$

Note that by applying transformation $\bar{s} = -s$ to (2) we obtain relation (7), suitable for designing orthogonal Legendre type filters, given in Figure 2.

Labels in the figure have the following meaning: $h(t)$ represent the step input signal and functions $\varphi_i(t)$ are

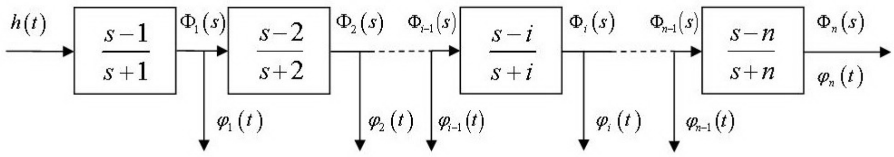

Figure 2. Legendre type orthogonal filter.

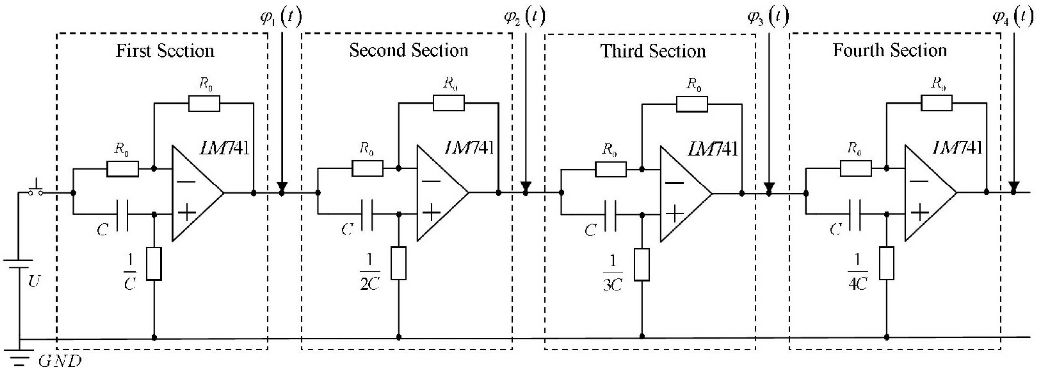

Figure 3. Practical realization of Legendre type orthogonal filter.

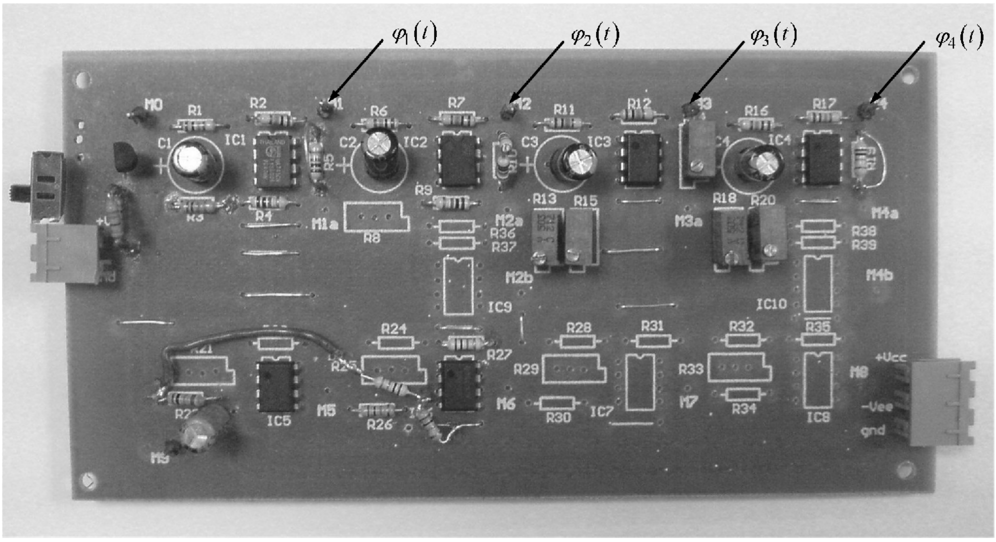

Figure 4. Legendre orthogonal filter, printed circuit board.

inverse Laplace transforms of the functions $\Phi_i(s)$. Functions sequence $\varphi_i(t)$ represents the series of Legendre exponential functions orthogonal over interval (0, ∞) with the weight $w(t) = e^{-t}$.

So, the functions:

$$\varphi_k(t) = L^{-1}\left[W_k(s)\right] = L^{-1}\left[\frac{1}{s}\frac{\prod_{i=1}^{k}(s-i)}{\prod_{i=1}^{k}(s+i)}\right] \tag{8}$$

are orthogonal over (0, ∞), i.e.,

$$\int_0^{\infty}\varphi_i(t)\varphi_j(t)e^{-t}dt = \begin{Bmatrix} 0, & i \neq j \\ N_i \neq 0, & i = j \end{Bmatrix},\ t > 0 \tag{9}$$

where:

$$\varphi_i(t) = \frac{1}{i!}\sum_{j=0}^{i}(-1)^{i-j}\binom{i}{j}\frac{(i+j)!}{j!}e^{-jt} \tag{10}$$

with the first few members of the series:

$\varphi_0(t) = 1,$

$\varphi_1(t) = 2e^{-t} - 1,$

$\varphi_2(t) = 6e^{-2t} - 6e^{-t} + 1,$

$\varphi_3(t) = 20e^{-3t} - 30e^{-2t} + 12e^{-t} - 1, \ldots$

Obtained filter scheme given in Figure 2 is very simple and suitable for practical realization. We have designed, in our Laboratory for modeling, simulation, and systems control, printed circuit board for developed filter. Orthogonal Legendre type filter has the practical realization shown in Figure 3. This particular filter, developed for further experiments, consists of four sections with printed circuit board given in Figure 4. Signals recorded on realized orthogonal filter are presented in Figure 5.

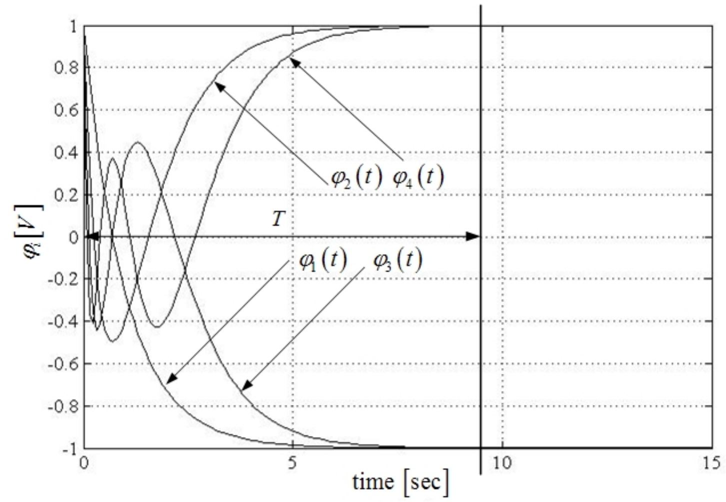

Figure 5. Signals recorded on realized orthogonal filter.

IV. CONTROL ALGORITHM – GRADIENT DETECTION BASED ON ORTHOGONAL FILTERS

Proposed control algorithm for antenna system positioning consists of two parts: gradient detection and movement organization toward extremum [2], [3], [19].

Block diagram of gradient detection system with orthogonal signals $\varphi_1(t),\varphi_2(t),\ldots,\varphi_n(t)$ is shown in Figure 6. These signals are exponential [17] and in this paper, they have been used for gradient detection. In order to effectively use the detection algorithm, it is necessary that these signals are continuously generated with a certain period of repetition (*T*). This period represents the time needed for signals $\varphi_i(t)$ to reach stationary state i.e. a constant value (see Figure 5). Inputs signals are labeled with $y_1^0, y_2^0,\ldots,y_n^0$. Orthogonal components $A_i\varphi_i(t)$ obtained from filter in Figure 4 are being added to the inputs signals:

$$
\begin{aligned}
y_1 &= y_1^0 + A_1\varphi_1(t)\\
y_2 &= y_2^0 + A_2\varphi_2(t)\\
&\vdots\\
y_n &= y_n^0 + A_n\varphi_n(t)
\end{aligned} \tag{11}
$$

Output signal from antenna system is a complex nonlinear function (in this case convex) of paraboloid type. This signal is led into gradient detector. Detector consists of *n* branches (the number of branches is equal to the number of input coordinates). Each branch consists of multipliers for field magnitude signal $F(y_1,y_2,\ldots,y_n)$ and corresponding orthogonal signals $\varphi_i(t)$. Obtained product then goes to the mean square element, which calculates the following relation:

$$
\overline{F(y_1,y_2,\ldots,y_n)\varphi_i(t)w(t)} = \int_0^T F\varphi_i(t)w(t)\,dt \tag{12}
$$

where $w(t)=e^{-t}$ represent the weight function.

After developing (12) into series and by substituting: $A_i\varphi_i(t)=\Delta y_i$, we obtain:

$$
\begin{aligned}
&F(y_1,y_2,\ldots,y_n) =\\
&= F\left(y_1^0+\Delta y_1, y_2^0+\Delta y_2,\ldots,y_n^0+\Delta y_n\right) =\\
&= F\left(y_1^0,y_2^0,\ldots,y_n^0\right) + \sum_{j=1}^{n}\left.\frac{\partial F}{\partial y_j}\right|_{y_j=y_{j0}}\Delta y_j +\\
&+\frac{1}{2!}\sum_{j_1}\sum_{j_2}\left.\frac{\partial^2 F}{\partial y_{j_1}\partial y_{j_2}}\right|_{y_j=y_{j0}}\Delta y_{j_1}\Delta y_{j_2}+\cdots\\
&\cdots+\frac{1}{n!}\sum_{j_1,j_2,\ldots j_k}\left.\frac{\partial^k F}{\partial y_{j_1}\partial y_{j_2}\cdots\partial y_{jk}}\right|_{y_j=y_{j0}}\Delta y_{j_1}\Delta y_{j_2}\cdots\Delta y_{jk}
\end{aligned} \tag{13}
$$

It should be noticed that for the function *F* gradient, the following relation is valid:

$$
gradF(y_1,y_2,\ldots,y_n) = \sum_{i=1}^{n}\frac{\partial F}{\partial y_i}\vec{i}_i \tag{14}
$$

where $\vec{i}_i$ represent unitary vector.

Equation (13) enables determining of gradient components $\partial F/\partial y_i$ based on orthogonal filters, which will be necessary for the realization of the gradient control algorithm.

After multiplying (13) with $\varphi_n(t)$ and applying the property of orthogonality for functions $\varphi_i(t)$, we have:

$$
\begin{aligned}
&\overline{F(y_1,y_2,\ldots,y_n)\varphi_n(t)w(t)} \approx\\
&\approx \sum_{j=1}^{n}\left.\frac{\partial F}{\partial y_j}\right|_{y_j=y_{j0}}\Delta y_j A_j\int_0^T\varphi_j(t)\varphi_n(t)w(t)\,dt =\\
&=\frac{\partial F}{\partial y_n}A_n\int_0^T\varphi_n(t)\varphi_n(t)w(t)\,dt = k_n\frac{\partial F}{\partial y_n}
\end{aligned} \tag{15}
$$

where:

$$
k_n = A_n\int_0^T\varphi_n(t)\varphi_n(t)w(t)\,dt \tag{16}
$$

and [6]:

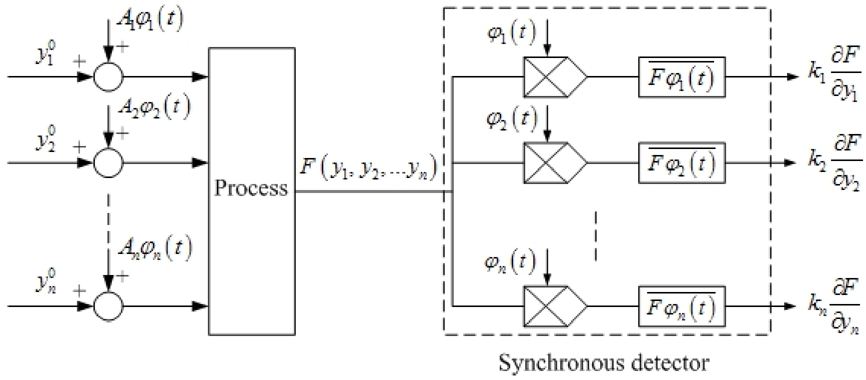

Figure 6. Gradient detection system.

$$k_n = \frac{1}{2n+1} \tag{17}$$

Now we denote function F gradient with G: ($G = gradF$), and normalized value of G with:

$$\boldsymbol{G}_{0j} = \frac{\boldsymbol{G}}{|G|} = \frac{\sum_i \frac{\partial F}{\partial y_i}\vec{i}_n}{\sqrt{\sum_i \left(\frac{\partial F}{\partial y_i}\right)^2}} \tag{18}$$

In this paper, target tracking in based on gradient method [2], [3], [19] and the following relation:

$$\boldsymbol{\theta}(j+1) = \boldsymbol{\theta}(j) - h(j)\boldsymbol{G}_{0j} \tag{19}$$

where $h(j)$ represents the weight function. $h(j)$ can be either constant or variable. Constant step is used when we want faster movements but with the disadvantage of oscillations appearing over extremes, near the movement ends. For the cases when we need more precise and accurate control of the antenna, we can use variable step, which decreases with reaching the extremes. In our case, there are two components of gradient:

$$\begin{aligned} \theta_a(j+1) &= \theta_a(j) - h_a(j)G_{0j} \\ \theta_e(j+1) &= \theta_e(j) - h_e(j)G_{0j} \end{aligned} \tag{20}$$

where θ_a and θ_e represent azimuth and elevation angle, respectively.

V. EXPERIMENTAL RESULTS

Antenna system, as a controlled object, operates with two coordinates: azimuth angle θ_a and elevation angle θ_e. Values of these two coordinates completely determine antenna direction toward source of radiation. Magnitude of the radiation source field $F(\theta_a, \theta_e)$ depends on angles θ_a and θ_e. In the case of two coordinates, orthogonal Legendre filter has the configuration given in Figure 3 with two sections.

In this case, functions $\varphi_1(t)$ and $\varphi_2(t)$ in Figure 2 are mutually orthogonal with weight function e^{-t}. According to (14), gradient is: $gradF(\theta_a, \theta_e) = \frac{\partial F}{\partial \theta_a}\vec{i}_1 + \frac{\partial F}{\partial \theta_e}\vec{i}_2$. Block diagram of the antenna system is shown in Figure 7 and the experimental system, developed in our laboratory is given in Figure 8.

Angles θ_a and θ_e are adjusted via two step motors M1 and M2 (motors are 86BYG-NEMA34 and ROB-09238 with the following characteristics: step angle – 1.2 deg; steps per revolution – 300; angular accuracy – ±3%; phases – 2; operating temperature – -20 to 40 °C; rated voltage – 12V; rated current – 0.33A; holding torque – 2.3 kg*cm). It should be noticed that with these motors [20], the number of steps is proportional to the control voltages. Components of electromagnetic field gradient are detected using synchronous detector (see Figure 6). Each motor is controlled by appropriate output from gradient detector ($k_1\frac{\partial F}{\partial \theta_a}, k_2\frac{\partial F}{\partial \theta_e}$). In order to control real object, these signals are amplified (K in Figure 7) and then led to the reductors for adjustment of angles θ_a and θ_e. Motors M1 and M2 are controlled separately, but complete system turns toward maximum field change i.e., in direction of electromagnetic field gradient. Described control algorithm operates until maximum field magnitude is reached and antenna is turned toward radiation source. Proposed method enables optimal tracking of moving sources and can be also used with mounted antennas on vehicles.

Figure 9 shows antenna movement trajectory in (θ_a, θ_e) plane (in degrees) toward field maximum (F_{max}) from arbitrary starting antenna position (x_0) and illustrates the main aspects of the proposed control. Figure 9 also shows two equidistant lines with equal field magnitudes and some labeled points for a better explanation of the control method described earlier.

In order to verify our control method, we have performed three experiments with different controllers and done comparative analysis. The goal was for antenna to track the field maximum as accurate as possible. In these experiments, we have used a simple light source (100W lamp) and photo sensor, but the method is the same for any other field source. Experimental test source trajectory in (θ_a, θ_e) plane is given in Figure 10 for duration of 20 seconds. We can more easily follow the results if we decompose movement trajectory into separate components for azimuth and elevation in time domain as shown in Figures 11 and 12. During experiments, antenna system was positioned on the table, so the elevation angle is limited from 0 to 90 degrees and azimuth angle takes values for full circle.

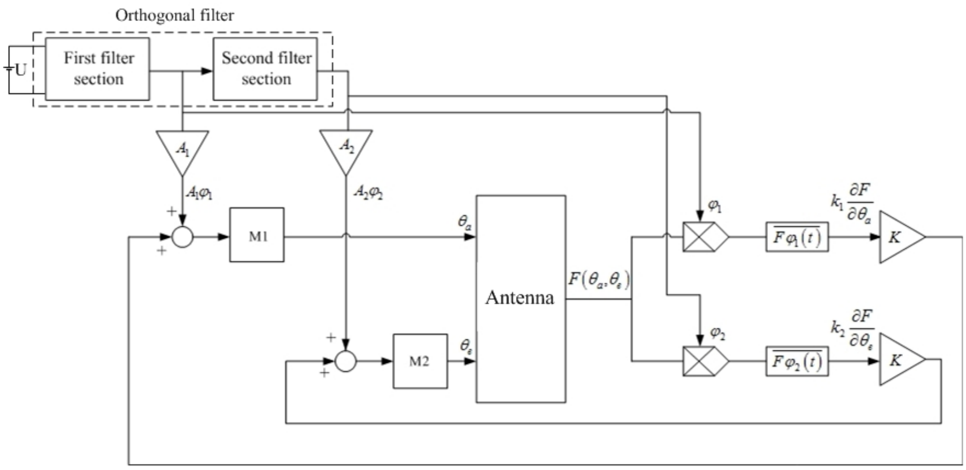

Figure 7. Block diagram of antenna system.

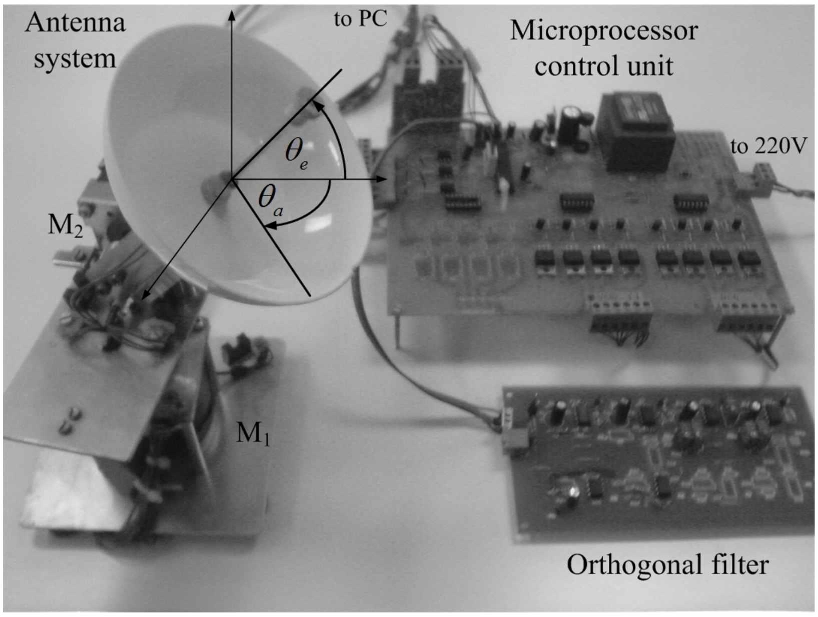

Figure 8. Antenna system, laboratory setup.

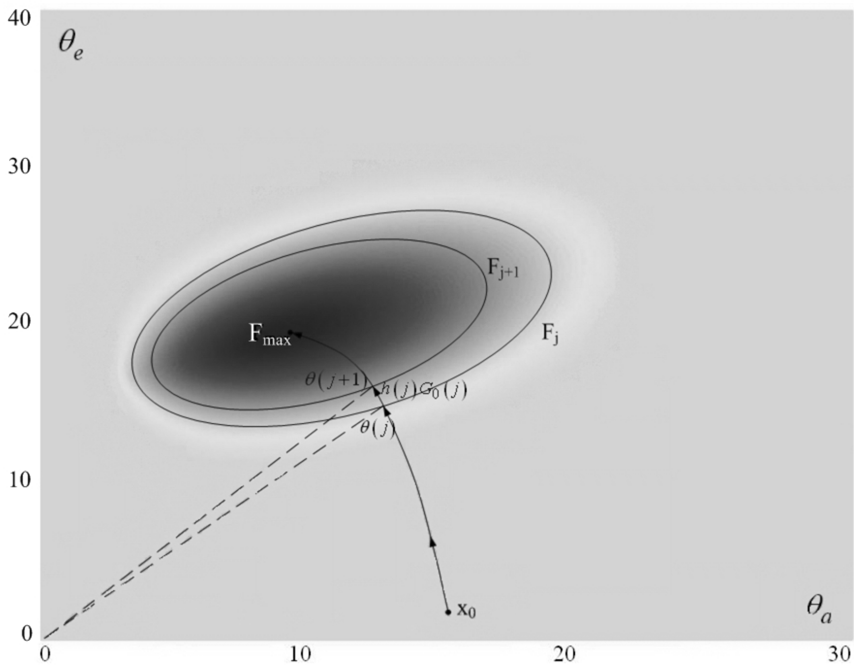

Figure 9. Antenna movement trajectory.

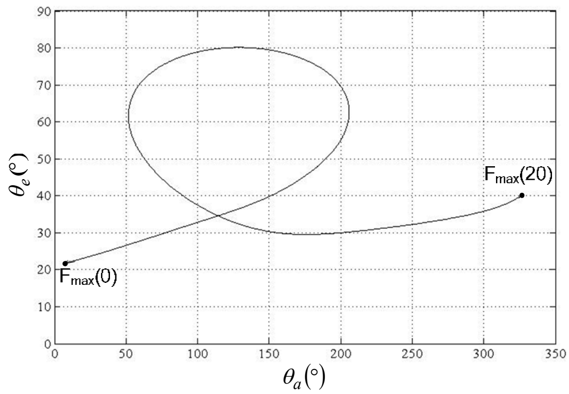

Figure 10. Trajectory of the field maximum (F_{max}).

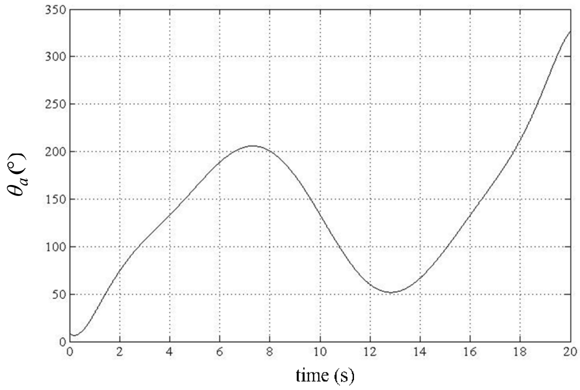

Figure 11. Azimuth component in time domain.

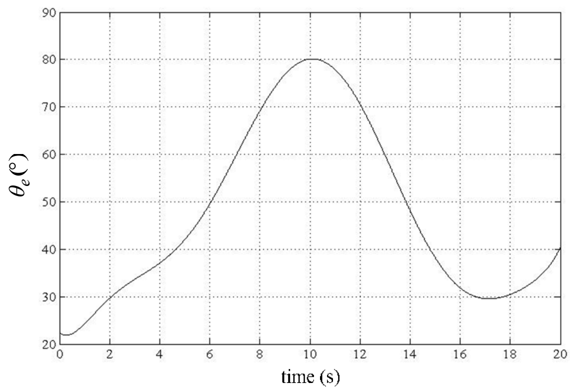

Figure 12. Elevation component in time domain.

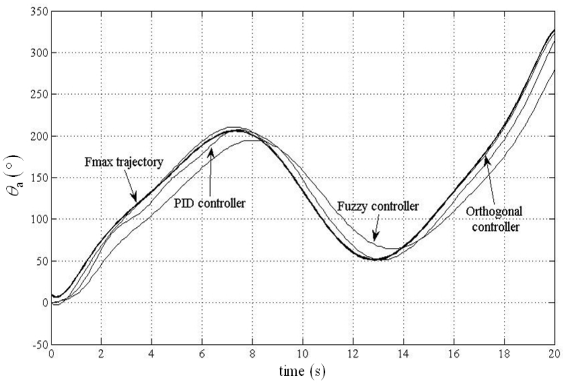

Figure 13. Azimuth tracking results for different controllers employed.

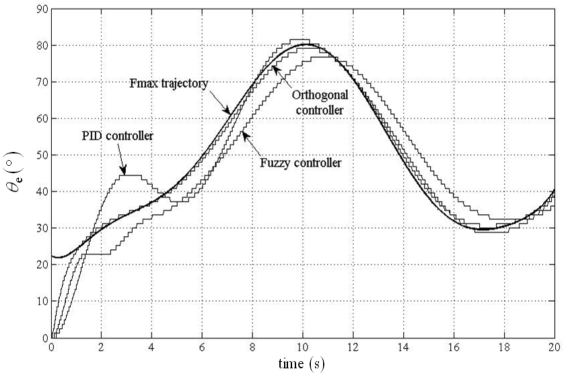

Figure 14. Elevation tracking results for different controllers employed.

In our experiments, we have employed three different controllers: PID controller, fuzzy controller and orthogonal controller described in this paper. Controllers were designed and adjusted experimentally without knowing the transfer function of the controlled object (the antenna). PID controller was adjusted using Ziegler-Nichols method and fuzzy controller was PD like with error and error difference as inputs and motors step commands as outputs. Experimental results for antenna tracking in all three cases are given in Figures 13 and 14. We can see from the figures that our control method has the best tracking results. Fuzzy controller is much slower and PID controller causes overshoots during the rapid changes of target position. We can calculate performance indexes as mean squared errors using the relation:

$$J = \frac{1}{2}\left(\frac{1}{T}\int_0^T \left(y_{ta} - y_{aa}\right)^2 dt + \frac{1}{T}\int_0^T \left(y_{te} - y_{ae}\right)^2 dt\right) \tag{21}$$

where T represents experiment duration ($T=20s$), y_{ta} and y_{te} azimuth and elevation components of target (field maximum) positions, y_{aa} and y_{ae} azimuth and elevation components of antenna position for different controllers applied. Calculated performances for orthogonal, PID and fuzzy controllers are J_{ort}=8.491, J_{pid}=17.432, and J_{fuz}=37.833.

VI. CONCLUSION

In this paper, we have proposed a new method for antenna system control, suitable for moving target tracking, based on orthogonal filters. Advantage of this approach is in application of new class of orthogonal filters based on Legendre functions for easy determining electromagnetic field magnitude gradient. Realization of these filters is very simple and they are very fast, robust and precise. They are also very convenient for application of gradient methods in optimization and adaptation problems because of their feature to speed up existing (classical) control algorithms. Based on proposed methodology, we have practically realized all the necessary components and the antenna system as a whole. Laboratory experiments and comparative study have validated our method for antenna system control in terms of efficiency, speed, and tracking accuracy.

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