Improved Nyquist Filters with a Transfer Characteristic Derived from a Staircase Characteristic Interpolated with Sine Functions

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***Abstract*—This paper investigates a novel approach for constructing a family of ISI-free pulses produced by improved Nyquist filters with a transfer characteristic derived from an ideal staircase frequency characteristic using interpolation with sine functions. They equal or outperform some recently proposed pulses in terms of ISI performance in the presence of sampling errors. The results for error probability outperform the 4th degree polynomial pulse for a reasonable number of interpolation intervals. The proposed pulses were also investigated for OFDM use including DVB systems in order to reduce their sensitivity to frequency offset. The results presented in this paper equal those of recently found pulses in terms of intercarrier interference (ICI) power.**

***Index Terms*—DVB, Nyquist filter, Improved impulse response, Inter-Symbol-Interference, OFDM**

I. INTRODUCTION

Recent works [1], [13], [5] have reported and examined new families of pulses which are inter-symbol interference (ISI)-free and that asymptotically decay as t^{-3}, t^{-2} and t^{-k} for any integer value of k, respectively [5].

Recently, new improved Nyquist pulses were introduced [3], [9], [4] and [11], showing smaller maximal distortion, a more open eye diagram at the receiver and a smaller error rate in the presence of synchronization errors, i.e. sampling the received signal with an offset with regard to ideal sampling instants.

In [1], [13] and [5], G(f) was chosen to be described by a particular waveform in the frequency interval $B(1-\alpha) \leq |f| \leq B$ in order to transfer spectral energy to the higher frequencies. This results in a pulse that decreases asymptotically as t^{-2}, as compared with t^{-3} for the RC pulse.

The resulted time domain pulses, denoted in the sequel improved Nyquist pulses have a decreased size first side lobe at the expense of a slight increase in the size of remaining lobes.

In a real transmission the received signal is sampled with an offset with regard to ideal sampling instants and due to the decrease of the first side lobe this results in a decrease of the intersymbol interference and a smaller value for the probability of error.

In order to have an asymptotic decay rate ADR of the impulse response of t^{-2}, the first derivative of the transfer function should have one or more finite amplitude discontinuities.

II. STAIRCASE CHARACTERISTIC

In a previous paper [9] a low-pass filter with ideal piece-wise rectangular transfer characteristic showing odd symmetry about the corresponding ideally band-limited cut-off frequency was proposed and investigated.

Here one started from an ideal frequency characteristic composed of rectangles of equal width and obeying an odd-symmetry law, denoted as $X_k(f)$.

This frequency characteristic is denoted as staircase characteristic and is defined for positive frequencies by equation (1) and is illustrated in Figure 1 for a particular case.

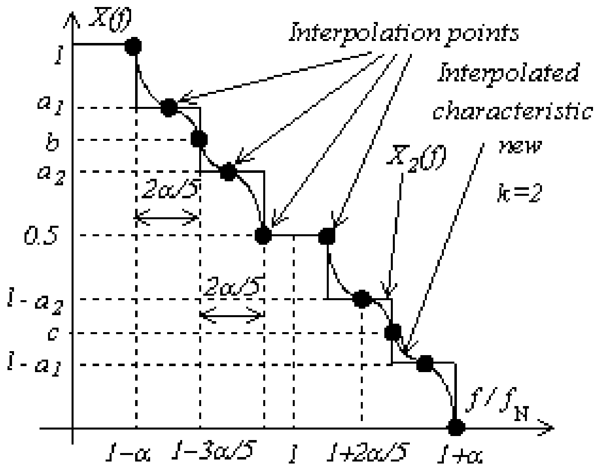

Figure 1. Frequency characteristic number 1 (i = 1).

We shall consider as an example a staircase transfer function of the improved Nyquist filter $X_k(f)$ defined by two parameters a_1 and a_2 (k = 2)

$$X_2(f) = F(\alpha, a_1, a_2) \tag{1}$$

The frequency characteristic of the Nyquist filter with a staircase transfer function is defined by relation (2) and is dependent on the parameters α, a_1 and a_2.

The transfer function $X_2(f)$ is constant over seven intervals in the filter bandwidth for positive frequency, as shown in figure 2 for the particular case α = 0.5.

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$$X_2(f)=\begin{cases}1, & 0<f<1-\alpha\\ a_1, & 1-\alpha<f<1-3\alpha/5\\ a_2, & 1-3\alpha/5<f<1-\alpha/5\\ 0.5, & 1-\alpha/5<f<1+\alpha/5\\ 1-a_1, & 1+\alpha/5<f<1+3\alpha/5\\ 1-a_2, & 1+3\alpha/5<f<1+\alpha\\ 0, & 1+\alpha<f\end{cases} \tag{2}$$

Because the ideal *staircase* frequency characteristic is physically unrealizable, several interpolation techniques can be used in order that the filter transfer characteristic should be physically implemented.

The interpolation points are marked with circles. Several choices were made regarding the frequency characteristic implementation, as shown in figures 2 and 3 for α = 0.5, denoted as 1 and 2, respectively.

III. INTERPOLATION METHOD

The method for constructing the filter characteristics proposed and investigated here uses sine functions approximation.

We approximated the *staircase* characteristic by n pieces of sine functions that link the interpolation points chosen on the staircase characteristic.

The idea is to link two points of coordinates (x_1, y_1) and (x_2, y_2) by a piece of a sine function, as shown in Figure 2 for a piece-wise rectangular transfer characteristic composed of 7 rectangles (k = 2), or using another arrangement.

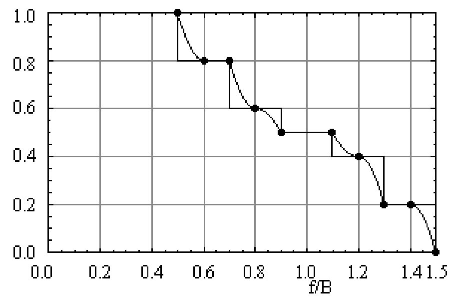

Figure 2. Frequency characteristic number 2 (α = 0.5).

In a previous paper [9] it was shown that if the frequency characteristic is flat around Nyquist frequency, this results in improved performance regarding error probability, when the impulse response is sampled with a timing offset, as it happens in real life.

So, we chose that the sine function should have a phase of 3π/2 at the point defined by the coordinates (x_2, y_2).

There are 3 parameters that can be used in order to adjust the sine function to the imposed task, namely frequency, amplitude and phase.

Let us denote by m the frequency of sine function that passes through the points defined by (x_1, y_1) and (x_2, y_2) where m is not restricted to be an integer, by b the amplitude of sine function, by a the offset on vertical axis and by φ the phase.

So,

$$f(x)=a+b\cdot\sin(2\pi mx+\varphi) \tag{3}$$

So, it is obvious that

$$f(x_1)=y_1 \tag{4}$$

$$f(x_2)=y_2 \tag{5}$$

Imposing that the sine function should be maximal flat in the interpolation point, so that the limit to the right of the derivative of sine function is zero, we get relation (6):

$$f(x_2)=a-b=y_2 \;\rightarrow\; b=a-y_2 \tag{6}$$

Also

$$2\pi mx_2+\varphi=3\pi/2 \;\rightarrow\; \varphi=3\pi/2-2\pi mx_2 \tag{7}$$

This condition was imposed in order for the interpolated characteristic to present discontinuities of the first derivative at the interpolation points. Also, the interpolated characteristic is closer to the ideal one.

As the pieces of sine functions used for interpolations do not encompass an integer number of half-periods, the value of the first derivative evaluated at the other end of interpolation interval will not be zero. This results in an impulse response that decays as t^{-2}, as shown in [4].

$$f(x)=a+b\cdot\sin[2\pi m(x-x_2)+3\pi/2]=y_1 \tag{8}$$

or

$$f(x)=a+(a-y_2)\cdot\sin\left[2\pi m(x-x_2)+3\pi/2\right] \tag{9}$$

Then evaluating f(x) at $x=x_1$

$$f(x_1)=a+(a-y_2)\cdot\sin\left[2\pi m(x_1-x_2)+3\pi/2\right]=y_1 \tag{10}$$

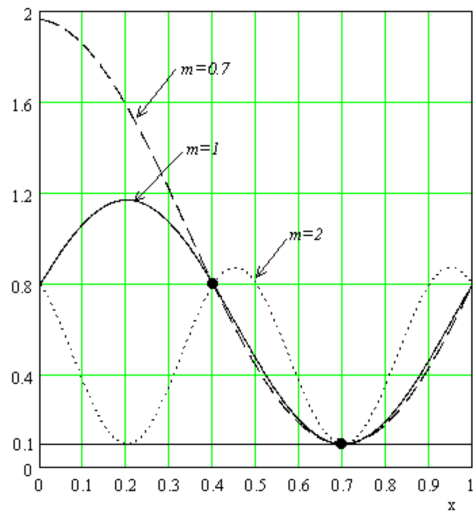

Figure 3. Sine functions that pass through the points (0.4, 0.8) and (0.7, 0.1) for three values of m.

Solving eq.(3) for *a*, we get eq. (11) and (12):

$$a=\frac{y_1+y_2\cdot\sin\left[2\pi m(x_1-x_2)+3\pi/2\right]}{1+\sin\left[2\pi m(x_1-x_2)+3\pi/2\right]} \tag{11}$$

and

$$f(x)=\frac{y_1+y_2\cdot\sin\left[2\pi m(x_1-x_2)+3\pi/2\right]}{1+\sin\left[2\pi m(x_1-x_2)+3\pi/2\right]}+$$

$$+\left(\frac{y_1+y_2\cdot\sin\left[2\pi m(x_1-x_2)+3\pi/2\right]}{1+\sin\left[2\pi m(x_1-x_2)+3\pi/2\right]}-y_2\right)\cdot \tag{12}$$

$$\cdot\sin\left[2\pi m(x-x_2)+3\pi/2\right]$$

An example of sine functions that pass through two particular points of coordinates (0.4, 0.8) and (0.7, 0.1) for three values of m, namely 0.7, 1 and 2 and have a phase of $3\pi/2$ at the point defined by the co-ordinates (0.7, 0.1) is illustrated in Fig. 3.

TABLE I.
COORDINATES OF INTERPOLATION POINTS ($\kappa = 2$)

Ends of interpolation interval	Interval width	Coordinates of interpolation points
No interpolation $(0,\ 1-\alpha)$	$1-\alpha$	$(0,\ 1-\alpha)$, $(1-\alpha,\ 1)$
$(1-\alpha,\ 1-4\alpha/5)$	$\alpha/5$	$(1-\alpha,\ 1)$, $(1-4\alpha/5,\ a_1)$
$(1-4\alpha/5,\ 1-3\alpha/5)$	$\alpha/5$	$(1-4\alpha/5,\ a_1)$ $(1-3\alpha/5,\ b)$
$(1-3\alpha/5, 1-2\alpha/5)$	$\alpha/5$	$(1-3\alpha/5,\ b)$ $(1-2\alpha/5,\ a_2)$
$(1-2\alpha/5, 1-\alpha/5)$	$\alpha/5$	$(1-2\alpha/5,\ a_2)$ $(1-\alpha/5,\ 0.5)$
No interpolation $(1-\alpha/5, 1+\alpha/5)$	$2\alpha/5$	$(1-\alpha/5, 0.5)$, $(1+\alpha/5, 0.5)$
$(1+\alpha/5, 1+2\alpha/5)$	$\alpha/5$	$(1+\alpha/5, 0.5)$, $(1+2\alpha/5, 1-a_2)$
$(1+2\alpha/5, 1+3\alpha/5)$	$\alpha/5$	$(1+2\alpha/5, 1-a_2)$, $(1+3\alpha/5, c)$
$(1+3\alpha/5, 1+4\alpha/5)$	$\alpha/5$	$(1+3\alpha/5, c)$, $(1+4\alpha/5, 1-a_1)$
$(1+4\alpha/5, 1+\alpha/5)$	$\alpha/5$	$(1+4\alpha/5, 1-a_1)$, $(1+\alpha,\ 0)$

where $b = \dfrac{a_1 + a_2}{2}$ and $c = \dfrac{2 - a_1 - a_2}{2}$.

The interpolation will result in a new characteristic that is close to the staircase characteristic. This behavior will be reflected in the results obtained in terms of error probability. Increasing the number k of interpolation points will also determine the shape of the new characteristic to be closer to the ideal *staircase* characteristic.

The same is valid for decreasing m. For k = 2, the pair of interpolation points for sine functions are illustrated in Tables 1 and 2.

As an example we shall write the equation of a sine function of phase $3\pi/2$ at $(1-4\alpha/5, a_1)$ that passes through the points of coordinates $(1-\alpha,\ 1)$ and $(1-4\alpha/5, a_1)$.

$$y_1 = 1,\quad y_2 = a_1 \qquad x_1 = 1-\alpha,\quad x_2 = 1-4\alpha/5$$

$$\begin{aligned} f_1(x) &= \frac{1 + a_1 \cdot \sin\left[-2\pi m\alpha/5 + 3\pi/2\right]}{1 + \sin\left[-2\pi m\alpha/5 + 3\pi/2\right]} + \\ &+ \left(\frac{1 + a_1 \cdot \sin\left[-2\pi m\alpha/5 + 3\pi/2\right]}{1 + \sin\left[-2\pi m\alpha/5 + 3\pi/2\right]} - a_1 \right) \cdot \\ &\cdot \sin\left[2\pi m(x - 1 + 4\alpha/5) + 3\pi/2\right] \end{aligned} \tag{13}$$

TABLE II.
COORDINATES OF INTERPOLATION POINTS ($\kappa = 2$)

Ends of interpolation interval	Interval width	Coordinates of interpolation points
No interpolation $(0,\ 1-\alpha)$	$1-\alpha$	$(0,\ 1-\alpha)$, $(1-\alpha,\ 1)$
$(1-\alpha,\ 1-4\alpha/5)$	$\alpha/5$	$(1-\alpha,\ 1)$, $(1-4\alpha/5,\ a_1)$
No interpolation $(1-4\alpha/5,\ 1-3\alpha/5)$	$\alpha/5$	$(1-4\alpha/5,\ a_1)$, $(1-3\alpha/5,\ a_1)$
$(1-3\alpha/5,\ 1-2\alpha/5)$	$\alpha/5$	$(1-3\alpha/5,\ a_1)$, $(1-2\alpha/5,\ a_2)$
$(1-2\alpha/5,\ 1-\alpha/5)$	$\alpha/5$	$(1-2\alpha/5,\ a_2)$, $(1-\alpha/5,\ 0.5)$
No interpolation $(1-\alpha/3,\ 1+\alpha/3)$	$2\alpha/5$	$(1-\alpha/5,\ 0.5)$, $(1+\alpha/5,\ 0.5)$
$(1+\alpha/5,\ 1+2\alpha/5)$	$\alpha/5$	$(1+\alpha/5,\ 0.5)$, $(1+2\alpha/5,\ 1-a_2)$
$(1+2\alpha/5,\ 1+3\alpha/5)$	$\alpha/5$	$(1+2\alpha/5,\ 1-a_2)$, $(1+3\alpha/5,\ 1-a_1)$
No interpolation $(1+3\alpha/5,\ 1+4\alpha/5)$	$\alpha/5$	$(1+3\alpha/5,\ 1-a_1)$, $(1+4\alpha/5,\ 1-a_1)$
$(1+4\alpha/5,\ 1+\alpha)$	$\alpha/5$	$(1+4\alpha/5,\ 1-a_1)$, $(1+\alpha,\ 0)$

The impulse response p(t) is obtained by performing the inverse Fourier transform of several pieces of sine function that pass through successive interpolation points.

For instance, in the interval $(1-\alpha,\ 1-4\alpha/5)$, the associated piece of sine function is given by rel. (13), and its contribution to the impulse response p(t) is eq. (14):

$$p_1(t) = \int_{1-\alpha}^{1-4\alpha/5} f_1(x) e^{j2\pi xt} dx \tag{14}$$

Performing the inverse Fourier transform results in eq. (15):

$$\begin{aligned} p_1(t) &= \frac{1}{8\pi t(t^2 - m^2)} \cdot \left(\operatorname{cosec}\left[\frac{\pi m\alpha}{5}\right]^2 \cdot 2 \cdot \left(-m^2 + a_1 t^2 + \right.\right. \\ &\left. + a_1(m^2 - t^2) \cdot \cos\left[\frac{2\pi m\alpha}{5}\right] \right) \cdot \sin\left[\frac{2}{5} \cdot (5 - 4\alpha)\pi t\right] + \\ &+ 2(m^2 - t^2) \cdot \left(a_1 \cdot \cos\left[\frac{2\pi m\alpha}{5}\right] - 1 \right) \cdot \sin\left[2(\alpha - 1)\pi t\right] - \\ &- (a_1 - 1) \cdot t \cdot (m+t) \cdot \sin\left[\frac{2\pi}{5}(\alpha(m\text{-}5t) + 5t)\right] + \\ &\left. + (m - t)\sin\left[\frac{2\alpha m\pi}{3} + 2(\alpha - 1)\pi t\right] \right. \end{aligned} \tag{15}$$

Also, for the interval $(1-\alpha/5, 1+\alpha/5)$, as f(x) = 0.5 we get eq. (16):

$$p_4(t) = \frac{\sin\left[2(1+\frac{\alpha}{5})\pi t\right] - \sin\left[2\left(1-\frac{\alpha}{5}\right)\pi t\right]}{4\pi t} \tag{16}$$

The impulse response $h_i(t)$ results from a sum of ten Fourier transforms, for both cases i = 1 and i = 2, and suffers a time scaling, $t \rightarrow t/2$ taking into account the constraints imposed by Nyquist I criterion for ISI-free signaling.

$$h_i(t) = \sum_{k=1}^{10} p_k(t) \tag{17}$$

The impulse response $h_2(t)$ is illustrated in Fig. 4 for $\alpha = 0.5$ together with impulse response g(t) defined in [4] by $a_2 = 25$, $a_3 = -64$, $a_4 = 55$ for *poly* pulse.

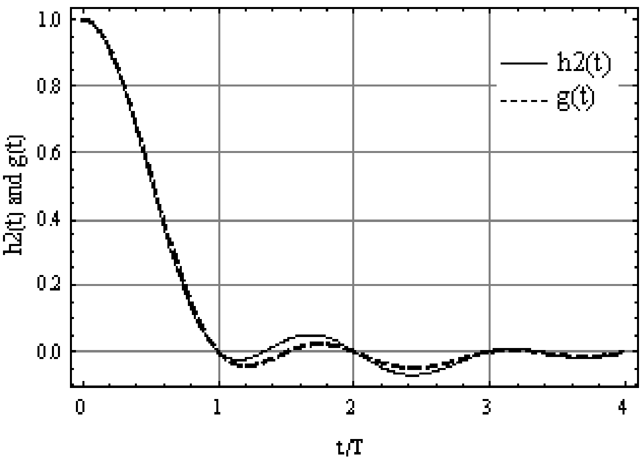

Figure 4. Impulse responses of new filter (i = 2, $a_1 = 0.68$, $a_2 = 0.6$) and *poly* filter [4] for $\alpha = 0.5$.

The impulse responses $h_1(t)$ and $h_2(t)$ are quite similar, and show no significant difference.

The optimal value of parameters a_1, a_2 were determined for the piece-wise rectangular improved Nyquist characteristic in a previous paper [9] and are presented in Tables 3 to 5, together with the values of the error probability for *poly* [5] characteristic defined by a_2, a_3 and a_4.

We worked with k = 2, a total number of intervals equal to 10 and a number of interpolation intervals 8 and 6, respectively, as illustrated in figures 1 and 2 and Tables 1 and 2.

Performing inverse Fourier transforms on the pieces of sine functions defined on 8 intervals plus 2 rectangular functions defined in Table 1 and 2 and summing all the contributions resulted in the impulse response h(t).

This is illustrated in figures 4, 5 and 6 together with poly pulse, taken as a reference.

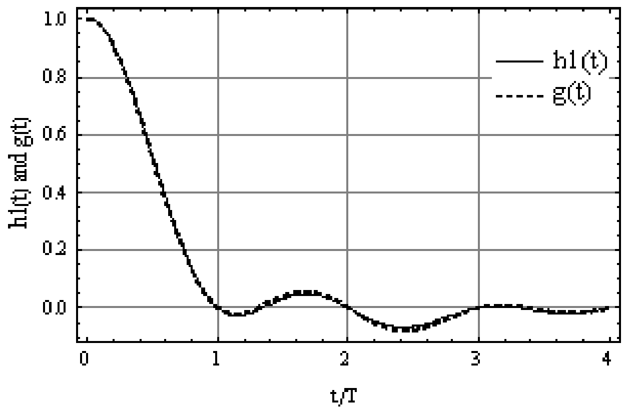

Figure 5. Impulse responses of *poly* and new filter (i = 1, $a_1 = 0.69$, $a_2 = 0.62$) for $\alpha = 0.35$.

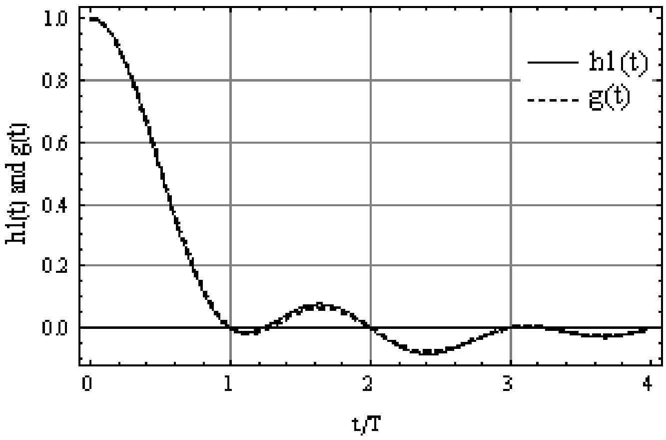

Figure 6. Impulse responses of *poly* and new filter (i = 1, $a_1 = 0.62$, $a_2 = 0.58$) for $\alpha = 0.25$.

IV. OFDM USE

The OFDM technique is very sensitive to carrier frequency offset caused by the jitter of carrier wave and phase errors between the transmitter and receiver. In the sequel we present and investigate the use of the new ISI-free pulses derived above in a 64-carrier OFDM system.

The complex envelope of one radio frequency (RF) N-subcarrier OFDM block with pulse-shaping [11] is expressed as:

$$x(t) = e^{j2\pi f_c t} \sum_{k=0}^{N-1} a_k p(t) e^{j2\pi f_k t} \tag{18}$$

where: $j = \sqrt{-1}$, f_C is the carrier frequency, f_k is the subcarrier frequency of the k-th subcarrier, $p(t)$ is the time-limited pulse shaping function and a_k is the data symbol transmitted on the k-th subcarrier and has mean zero and normalized average symbol energy; data symbols are uncorrelated.

Frequency offset, $\Delta f\ (\Delta f \geq 0)$, and phase error θ, are introduced during transmission because of channel distortion or receiver crystal oscillator inaccuracy.

The average ICI power, averaged across different sequences [11] is:

$$\overline{\sigma^{m}_{ICI}} = \sum_{\substack{k \neq m \\ k=0}}^{N-1} \left| P\left(\frac{k-m}{T}\right) + \Delta f \right|^{2} \tag{19}$$

The average ICI power depends not only on the desired symbol location m, and the transmitted symbol sequence, but also on the pulse-shaping function at the frequencies $\left(\left((k-m)/T\right)+\Delta f\right)$, $k \neq m$, where k = 0, 1, …, N -1 and the number N of subcarriers.

The ratio of average signal power to average ICI power is denoted as SIR and expressed in equation (20).

$$SIR = \left|P(\Delta F)\right|^{2} / \sum_{\substack{k \neq m \\ k=0}}^{N-1} \left|P((k-m)/T) + \Delta f\right|^{2} \tag{20}$$

Relation (20) was evaluated for a number N = 64 subcarriers both for the *poly* pulse [5] and the new pulse produced by sine interpolation, defined by k = 2. Almost perfect overlapping of the SIR characteristics was found, as inferred from figure 7, where SIR is represented as a function of frequency offset.

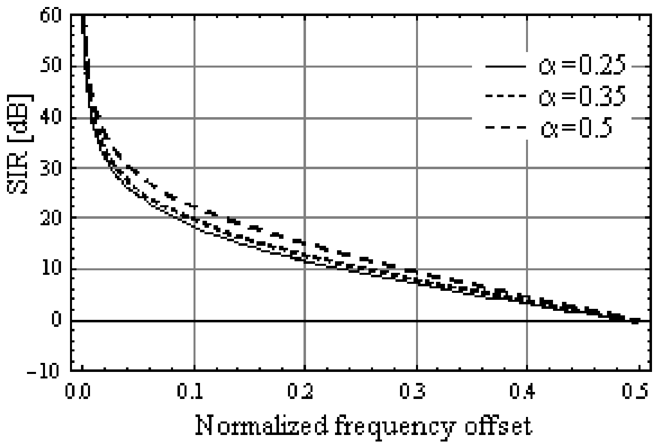

Figure 7. SIR for proposed pulses and *poly* pulse in a 64-subcarrier OFDM system.

V. SIMULATION RESULTS

In this section, the pulses produced by the improved Nyquist low-pass filters defined above are studied in terms of ISI error probability.

The probability of error measures the performances of the pulses regarding inter-symbol interferences and includes the effects of noise, synchronization error and distortion. The probability of error P_e was evaluated as in [2] using Fourier series.

$$P_e = \frac{1}{2} - \frac{2}{\pi} \sum_{\substack{m=1 \\ M odd}}^{M} \left(\frac{\exp\left(-m^2\omega^2/2\right)\sin\left(m\omega g_0\right)}{m} \right) \cdot \prod_{k=N_1}^{N_2} \cos\left(m\omega g_k\right) \tag{21}$$

Here M represents the number of coefficients considered in the approximate Fourier series of noise complementary distribution function; $\omega = 2\pi/T_f$ -angular frequency; T_f is the period used in the series; N_1 and N_2 represent the number of interfering symbols before and after the transmitted symbol; $g_k = p(t-kT)$ where p(t) is the pulse shape used and T is the bit interval.

The results are computed using $T_f = 40$ and M = 61 for N = 2^{10} interfering symbols and a signal to noise ratio SNR = 15 dB, for all the cases.

The values of the parameters $a_1, ..., a_k$, are determined so that the error probability should be minimized.

The results are reported in Tables 3 to 5 together with those for $X_2(f)$ and *poly* [5] pulse, taken as a reference.

TABLE III.
ISI ERROR PROBABILITY OF THE PROPOSED NYQUIST PULSES (K = 2) AND *POLY* FOR $N = 2^{10}$ INTERFERING SYMBOLS AND SNR = 15 DB

$X_2(f)$			$t/T_B = 0.05$		
			a_1	a_2	P_e
α = 0.25	$X_2(f)$		0.62	0.58	4.53042×10^{-8}
	new (k=2)	1	0.62	0.58	4.688×10^{-8}
		2	0,65	0.57	4.714×10^{-8}
	poly		(40, -100, 85)		4.734×10^{-8}
α = 0.35	$X_2(f)$		0.64	0.58	3.0597×10^{-8}
	new (k=2)	1	0.69	0.6	3.180×10^{-8}
		2	0.65	0.59	3.156×10^{-8}
	poly		(32, -80, 69)		3.290×10^{-8}
α = 0.5	$X_2(f)$		0.68	0.6	1.9494×10^{-8}
	new (k=2)	1	0.69	0.62	1.985×10^{-8}
		2	0.68	0.6	1.985×10^{-8}
	poly		(25, -64, 55)		2.057×10^{-8}

TABLE IV.
ISI ERROR PROBABILITY OF THE PROPOSED NYQUIST PULSES (K = 2) AND *POLY* FOR $N = 2^{10}$ INTERFERING SYMBOLS AND SNR = 15 DB

$X_2(f)$			$t/T_B = 0.1$		
			a_1	a_2	P_e
α = 0.25	$X_2(f)$		0.64	0.58	8.261×10^{-7}
	new (k=2)	1	0.65	0.58	8.735×10^{-7}
		2	0.66	0.59	8.715×10^{-7}
	poly		(40, -100, 85)		8.834×10^{-7}
α = 0.35	$X_2(f)$		0.68	0.6	3.568×10^{-7}
	new (k=2)	1	0.69	0.62	3.735×10^{-7}
		2	0.68	0.61	3.733×10^{-7}
	poly		(32, -80, 69)		3.839×10^{-7}
α = 0.5	$X_2(f)$		0.68	0.6	1.332×10^{-7}
	new (k=2)	1	0.71	0.63	1.318×10^{-7}
		2	0.7	0.62	1.319×10^{-7}
	poly		(25, -64, 55)		1.354×10^{-7}

TABLE V.
ISI ERROR PROBABILITY OF THE PROPOSED NYQUIST PULSES (K=2) AND *POLY* FOR $N = 2^{10}$ INTERFERING SYMBOLS AND SNR = 15 DB

$X_2(f)$			$t/T_B = 0.2$		
			a_1	a_2	P_e
$\alpha = 0.25$	$X_2(f)$		0.66	0.6	2.038×10^{-4}
	new (k=2)	1	0.67	0.60	2.166×10^{-4}
		2	0.67	0.61	2.166×10^{-4}
	poly		(40, -100, 85)		2.241×10^{-4}
$\alpha = 0.35$	$X_2(f)$		0.7	0.62	6.453×10^{-5}
	new (k=2)	1	0.72	0.63	6.560×10^{-5}
		2	0.71	0.63	6.563×10^{-5}
	poly		(32, -80, 69)		6.563×10^{-5}
$\alpha = 0.5$	$X_2(f)$		0.72	0.62	1.807×10^{-5}
	new (k=2)	1	0.755	0.65	1.575×10^{-5}
		2	0.75	0.65	1.590×10^{-5}
	poly		(25, -64, 55)		1.520×10^{-5}

VI. CONCLUSION

We proposed and evaluated a new type of a Nyquist transfer function that approximates an ideal staircase frequency characteristic with 6 levels using pieces of sine functions that pass through interpolation points in terms of inter-symbol interference.

We found the minimal values for error probability that are listed in Tables 3 to 5. The proposed pulses outperform the *poly* pulse for timing offset values $t/T_B = 0.05$ and $t/T_B = 0.1$ and are slightly outperformed by the *poly* pulse only for $\alpha = 0.5$ and $t/T_B = 0.2$.

Better values of the error probability could be obtained if the number k of rectangles in the *staircase* characteristic and the number n of interpolation points are increased, e.g. for $k > 2$ and $n > 10$.

The results for error probability in the presence of symbol timing error for the same excess bandwidth outperform the 4^{th} degree polynomial pulse [5] in most cases and are comparable with it for $k = 2$, $\alpha = 0.5$ and $t/T_B = 0.2$.

The new pulses and *poly* pulse and are equal in terms of SIR (ratio of average signal power to average ICI power) for OFDM use in the presence of frequency offset.

The new pulses are also suitable for use in DVB systems.

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